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THESIS

AN INTEGER PROGRAMMING MODEL FOR
NAVY'S MARITIME PATROL AVIATION FLEET

by

LCDR Robert W. Drash

September, 1990

Thesis Advisor:

R. Kevin Wood

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91-08091



91 8 16 025

REPORT DOCUMENTATION PAGE				
1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b RESTRICTIVE MARKINGS		
2a SECURITY CLASSIFICATION AUTHORITY		3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.		
2b DECLASSIFICATION/DOWNGRADING SCHEDULE				
4 PERFORMING ORGANIZATION REPORT NUMBER(S)		5 MONITORING ORGANIZATION REPORT NUMBER(S)		
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School	6b OFFICE SYMBOL (If applicable) 55	7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School		
6c ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000		7b ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000		
8a NAME OF FUNDING/SPONSORING ORGANIZATION	8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8c ADDRESS (City, State, and ZIP Code)		10 SOURCE OF FUNDING NUMBERS		
		Program Element No	Project No	Task No Work Unit Accession Number
11 TITLE (Include Security Classification) AN INTEGER PROGR. AMING MODEL FOR NAVY'S MARITIME PATROL AVIATION FLEET				
12 PERSONAL AUTHOR(S) Drash, Robert Wayne				
13a TYPE OF REPORT Master's Thesis	13b TIME COVERED From To	14 DATE OF REPORT (year, month, day) 90 September 09	15 PAGE COUNT 79	
16 SUPPLEMENTARY NOTATION The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government				
17 COSATI CODES		18 SUBJECT TERMS (continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUBGROUP		
		Optimization, P-3, Modernization, Integer Programming		
19 ABSTRACT (continue on reverse if necessary and identify by block number) This thesis details an integer programming model to aid in the modernization of the Navy's Maritime Patrol Aviation fleet. Over a user specified time horizon, the model provides a schedule for when to retire, perform avionics upgrades, or transfer current inventory aircraft from the USN to the USNR. Additionally, the model determines when to open a new aircraft production line and the number of aircraft to procure each year. The model optimizes the modernization schedule while taking into consideration required inventory, minimum required percentage of aircraft containing modern avionics, maximum desired mean aircraft age, budgetary limitations, and production line restrictions. The model minimizes the procurement, operating and maintenance costs using the X-System solver.				
20 DISTRIBUTION/AVAILABILITY OF ABSTRACT <input type="checkbox"/> UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS REPORT <input type="checkbox"/> LTIC USERS		21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a NAME OF RESPONSIBLE INDIVIDUAL R. Kevin Wood		22b TELEPHONE (Include Area code) 408-646 2523	22c OFFICE SYMBOL OA/Wd	

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An Integer Programming Model for
Navy's Maritime Patrol Aviation Fleet

by

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Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL

September 1990

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ABSTRACT

This thesis details an integer programming model to aid in the modernization of the Navy's Maritime Patrol Aviation fleet. Over a user specified time horizon, the model provides a schedule for when to retire, perform avionics upgrades, or transfer current inventory aircraft from the USN to the USNR. Additionally, the model determines when to open a new aircraft production line and the number of aircraft to procure each year. The model optimizes the modernization schedule while taking into consideration required inventory, minimum required percentage of aircraft containing modern avionics, maximum desired mean aircraft age, budgetary limitations, and production line restrictions. The model minimizes the procurement, operating, and maintenance costs using the X-System solver.



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DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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ACKNOWLEDGMENT

I would like to thank Professor Gerald Brown for providing the inspiration for my thesis, Professor Kevin Wood for assisting in the formulation and debugging, and Professor Richard Rosenthal for proofing the final product. I would like to express a special thanks to LTC Robert Clemence for assisting in the initial model formulation and providing an example EDITOR FORTRAN programs to guide my data entry, and to Mary Scott for educating me on the cost aspects of P-3 operations.

I would like to express my sincerest appreciation to my wife Vivian and sons Bobby and Andy. Without their support, understanding and sacrifices this thesis would not have been possible.

I. INTRODUCTION

The United States Navy's Maritime Patrol Aviation (MPA) fleet presently consists of 24 operational, 13 reserve and two training squadrons. The aircraft used in each squadron is the P-3 Orion manufactured by Lockheed Corp. As technological advances are made in the P-3 avionics as well as the potential adversary's capabilities, a modernization program must be initiated to ensure the MPA can accomplish its assigned mission. Additionally, in today's environment of defense cuts and force reductions, modifications to the present MPA force structure may be required. The problem is how to accomplish force modernization/reduction, while minimizing dollar expenditures. This thesis creates and solves a mathematical model for the modernization program.

In addition to modernizing the avionics to accomplish perceived missions in the twenty-first century, attention must be given to the average age of the MPA fleet. If modernization is attempted by solely retrofitting avionics into existing airframes, a point will occur in the future when existing aircraft will have to be retired after reaching the end of their service lives and new aircraft procured in order to maintain the required force levels. This defers the cost of new aircraft procurement into the future, but could result in a significant increase in dollar expenditures in those years. These expenditure increases may not be practical in an environment of reduced defense budgets. A more rational approach to the problem is to integrate avionics upgrades, new aircraft procurement, and aircraft retirements such that designated budgetary ceilings are not exceeded.

The MPA modernization problem is exacerbated by the age of the current fleet. A large portion of the MPA fleet was procured over 15 years ago (Figure 1). These aircraft are rapidly approaching the end of their service lives of 30 years. Indeed, 52% of the fleet was built prior to 1975 and 71% prior to 1980. These aircraft will need to be replaced prior to 2005 and 2010 respectively.

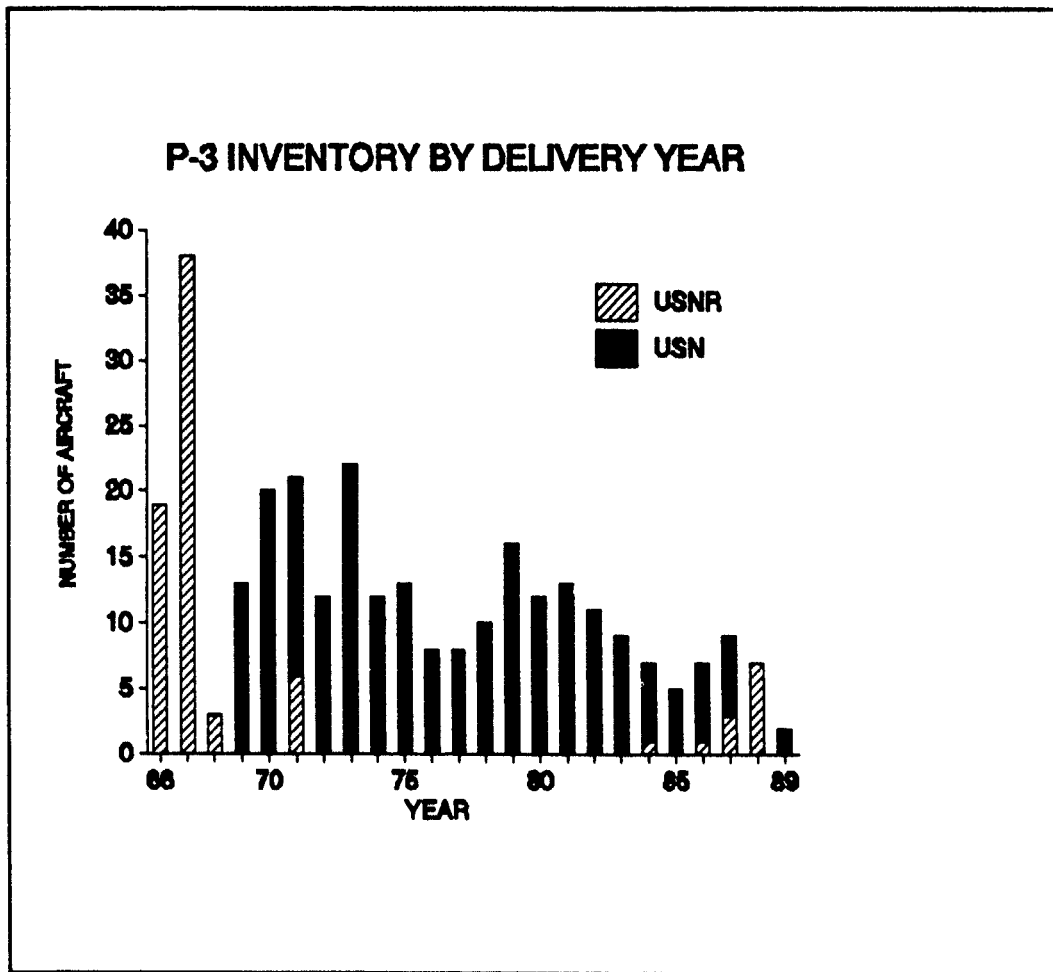


Figure 1. Current P-3 Inventory by Aircraft Delivery Year

There are presently four different model variants to the P-3 airframe: BMOD, C-U1, C-U2, C-U3. The avionics contained in each variant is different and therefore each variant has different capabilities. While each variant will have varying degrees of success against the most modern potential adversary, only the C-U3 is fully capable of handling this threat. As technological advances are made and new threats emerge, the C-U3's technological ability to counter these threats will diminish. Presently, C-U3s comprise approximately 44% of the MPA fleet (Figure 2). One problem facing the MPA fleet is how to ensure that adequate numbers of aircraft are available and capable of countering present and future threats.

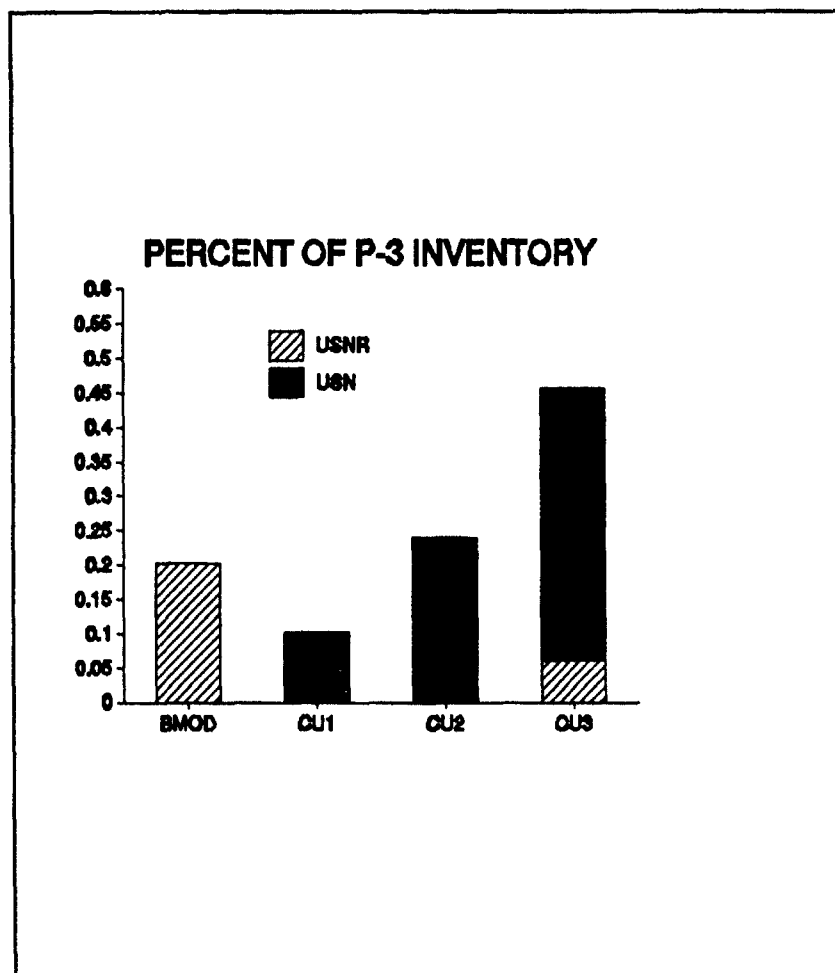


Figure 2. Current P-3 Inventory by Aircraft Model Type

The MPA modernization problem encompasses three primary concerns. First, the average age of the fleet must be maintained within a desired range. Second, the avionics must be upgraded to increase the percentage of the fleet able to counter future threats. Third, the previous two concerns must be accomplished while minimizing budgetary outlays. This thesis will detail a theoretical model for the MPA modernization program, implementation of the model and results of the model runs. The methodology has some precedence in the U.S. Army's helicopter modernization program [Ref. 1] begun in the late 1980s. That model, dubbed "Phoenix", helped solve the Army's helicopter modernization problem. It is used as a basic framework for the MPA modernization problem and is described below. The object of the MPA

modernization model is to determine the optimum time to procure new aircraft, and when to modernize, retire, or transfer existing aircraft from the United States Navy (USN) to the United States Naval Reserve (USNR). The planning horizon for the model is the time frame over which the model optimizes and is a parameter which can be varied by the user. The planning horizon should be long enough to ensure expenditures are not just delayed beyond the time horizon and thus not included in the final model solution. A discounted cost factor is added to the operating cost value for aircraft in the last year of the planning horizon. This factor will increment the cost of maintaining an aircraft in the fleet at the conclusion of the time horizon and thus make deferring the cost of replacing it less advantageous. For the purpose of this paper, the time horizon was chosen to be 20 years. The conclusion of this time horizon coincides with the year when 71% of the present fleet will have reached the end of their service lives and therefore be replaced. Due to computational difficulties to be discussed later, eight years was the maximum time horizon over which the model has been solved to date.

A. THE PHOENIX METHODOLOGY

The Phoenix model was developed jointly at the Naval Postgraduate School and the U.S. Army's Concepts Analysis Agency [Ref. 1]. The model addressed the modernization of the Army's helicopter fleet. The present fleet was aging rapidly, with a concurrent loss of high technology avionics. The model is a mixed-integer mathematical program which determines a schedule for the procurement of new and the refurbishment of old helicopters in order to meet the Army's long range numerical and percentage high technology goals, while keeping annual costs within limits.

There are a number of different missions which helicopters perform for the Army. Each mission requires a helicopter to perform certain tasks which may not be compatible with other missions, thereby requiring a specific model helicopter. Each helicopter model has its own production line, and future versions of the helicopter model, or a completely new model cannot be produced until the previous model line is closed.

Old helicopter airframes can be refurbished by undergoing a Service Life Extension Program (SLEP). A SLEP replaces fatigued airframe components and

installs up-to-date avionics. The airframe is essentially as good as new, and indeed is treated as such by issuance of a new Bureau Number (BUNO). If an airframe is not to undergo a SLEP, the only alternative available, at the expiration of its service life, is retirement.

The Phoenix model seeks to incorporate the above specifications for the desired planning horizon. The model establishes sets of constraints to ensure, for each year in the planning horizon:

- Required inventory level for each mission type is maintained,
- Desired minimum percentage of inventory contains high technology avionics,
- Maximum average fleet age is not exceeded, and
- Budgetary expenditures are maintained within a minimum/maximum window.

Additionally, constraints are created to ensure only one model is constructed on a given production line, and the line minimum/maximum yearly production limits are not exceeded. A complete description of the Phoenix model is contained in reference 1.

B. COMPARISON OF THE MPA PROBLEM TO PHOENIX

The Maritime Patrol Aviation (MPA) model has many similarities to the Phoenix model. The four sets of constraints in the Phoenix model described above are essentially directly applicable to the MPA model, except that there exists only one mission type in the MPA model. However, the MPA model diverges from the Phoenix model in several aspects.

There is currently no SLEP program for the P-3 fleet. Instead, the aircraft undergo periodic depot level maintenance which certifies the aircraft for a designated period of time, allowing it to continue to fly. The aircraft never become "new" as a "SLEP" aircraft does, and therefore must be retired upon reaching the expiration of their service life. Additionally, flight time accumulation records are maintained for P-3 aircraft, with aircraft being retired when maximum flight time limits are exceeded.

All avionics upgrades are accomplished through retrofits and new production aircraft. The retrofit and new aircraft production lines typically require the same

avionics and are therefore interrelated. The retrofit production line manufactures the new avionics, requiring it to remain open at least as long as the new aircraft production line.

This thesis will provide the user with a schedule for the avionics upgrades, transfers to the USNR, and retirements for existing fleet aircraft and the procurement of new production aircraft during the planning horizon.

II. MPA PROBLEM DESCRIPTION

The present MPA fleet is composed entirely of P-3 Orion aircraft. There are currently four variants of the P-3 airframe: BMOD, CU1, CU2, CU3. All of the variants have essentially the same airframe, with the exception of the BMOD which has a different sonobuoy delivery system. This airframe commonality among the "C" variants, allows older models to be upgraded to the most current variant by installing the latest avionics equipment. However, due to the airframe rework required to convert a BMOD to a "C" variant and the age of the airframe (all BMODs were built prior to 1970), there are no plans to install a new avionics suite in the BMODs.

In 1990, Boeing Corporation was awarded the contract to produce a new tactical station console and avionics upgrade for the P-3. The new tactical consoles will change the interior of the aircraft dramatically while incorporating new interfaces to allow future upgrades to be accomplished via software changes versus costly hardware modifications. This upgrade model, designated P-3U4 or simply "update 4", can be used in all previously produced P-3 airframes, except the BMOD, and is capable of countering potential threats of the 21st century.

In 1989 the sole producer of P-3s (Lockheed) shut down its production line permanently. The Navy determined the airframe could no longer accomplish the missions envisioned for the year 2000 and beyond. A completely new aircraft was designed and designated the P-7. The P-7 aircraft will incorporate the same tactical workstations and avionics which are being used in the update 4 upgrades. Therefore, while the update 4 and P-7 production lines are separate, they are interlocked in that the avionics/workstation line must remain open at least as long as the new airframe production line does.

An additional aspect to consider is the contractual agreements entered into by the government and the manufacturers. The government has agreed to purchase a minimum number of components by a certain year of the contract. For instance, the P-7 contract with Lockheed specifies the Navy will buy 8 aircraft before the fifth year

of the production campaign and 25 by the sixth year. If these contractual obligations were to be violated, a substantial penalty would be incurred.

Inherent in the specifications of aircraft and vehicles is a designed service life. The aircraft is expected to last to this point, while service beyond the designated service life requires extra inspections and possible airframe rework. Many aircraft have the capability of undergoing a Service Life Extension Program (SLEP) which essentially gives the airframe a "new" life. The airframe is "rebuilt" and sent back to the fleet as a "new" airframe with a new service life. The P-3 airframe does not presently have the option of undergoing a SLEP. P-3s do, however, undergo periodic Standard Depot Level Maintenance (SDLM) [Ref. 2]. A SDLM inspects and repairs corrosion and structural fatigue problems, but does not "rebuild" an airframe as a SLEP does. Additionally, avionic upgrades are not performed during a SDLM.

The SDLM process starts in the sixth year of operation with the initial Aircraft Service Period Adjustment (ASPA) [Ref. 3]. An ASPA determines whether there is sufficient corrosion or structural defects to warrant a SDLM. As long as an aircraft passes a yearly ASPA, it may continue to operate. However, once it fails an ASPA, it must undergo a SDLM. After completion of the SDLM, the aircraft is certified safe to fly for a period of 60 months following the first and second SDLM, 50 months for the third SDLM and 46 months for subsequent SDLMs [Ref 2:p. 1.2]. After this period, the ASPA process starts again.

Each SDLM varies in cost, depending on the amount of repairs required. However, for budgetary planning purposes, they are currently approximately \$580K, \$620K, and \$780K for the first three, respectively. When an aircraft reaches the end of its service life (approximately 30 years) or it exceeds the maximum flight time ceiling of 20,000 flight hours, it must undergo a mandatory SDLM (presently valued at \$1M). This SDLM cannot be deferred by an ASPA and will allow the airframe to remain in service for 40 more months. At that time another mandatory SDLM is preformed. This process could conceivably go on until the airframe is found to be structurally unsafe to fly. For the purposes of this model, it was assumed that all aircraft would be retired at or before forty years of service.

An additional aspect of the MPA problem is that the Navy MPA force is divided into two components: active (USN) and reserve (USNR). Each component has its own requirements for number of aircraft and guidelines for the average age and avionics capability required. An aircraft can be transferred to the USNR but not back to the USN. These requirements must be addressed as constraints in the model.

The final aspect of the MPA modernization problem concerns monetary outlays. In the present budgetary environment, large yearly expenditures are not seen in a favorable light. The MPA portion of the budget will probably be limited to a certain ceiling, unless justification can be provided for exceeding it.

III. MODEL DESCRIPTION

In order to realistically satisfy the requirements of the MPA modernization problem, a model must satisfy the following general categories of constraints:

- Inventory balance constraints to ensure each individual aircraft is counted only once.
- Required inventory constraints.
- Required degree of modern avionics (percentage of fleet having high technology avionics).
- Average age of fleet within acceptable limits.
- Maintain expenditures within budgetary limits.
- New aircraft and update 4 production restrictions.

Each of these sets of constraints will be discussed in detail below. In the ensuing discussion, it will be noted which constraints are considered "elastic" [Ref. 4]. Elasticity allows for a violation of a constraint by placing a per unit violation penalty in the objective function. This avoids having a model run simply to specify the problem is infeasible, which yields little information. Instead, the constraints which are violated are highlighted for the modeler, thereby providing insight into the problem and an indication that certain assumptions may have to be modified.

The following indices are used in the model:

- g cohort group (group of aircraft with similar characteristics: year group, flight time, model variant)
- a aircraft model type
- t time period (year)
- r service group attached to (USN,USNR)
- v production campaign start year
- w production campaign end year
- y y^{th} year of the production campaign

The following decision variables are used in the model:

I_{gtr}	Number of aircraft of model type a and associated with cohort group g which are contained in the inventory of service type r in year t.
U_{gt}	Number of aircraft associated with cohort group g which are updated in year t.
T_{gt}	Binary variable has a value of "1" when ALL aircraft associated with cohort group g are transferred to the reserves in year t.
R_{gt}	Binary variable has a value of "1" when ALL aircraft associated with cohort group g are retired to the reserves in year t.
P_{atk}	Binary variable has a value of "1" when k aircraft of model type a are produced in year t.

A convenient method of implementing the model would be to use general integer variables, combining all aircraft produced in the same year into one cohort group. The Army helicopter model used a methodology similar to this, except that the inventory variables were continuous rather than integer. In the MPA problem, however, a stipulation has been issued to maintain track of accumulated flight hours in order to ascertain when an aircraft exceeds its maximum flight time limit. Since the flight accumulation rates for the USN and USNR are different, this requires a determination of when an aircraft is transferred into the USNR. If all aircraft of a year group are combined, there would not be any method to differentiate when a specific aircraft exceeds its maximum limit. Therefore, each aircraft should be treated as a separate entity. This, however, creates an excessive number of variables. In order to maintain fidelity yet yield a model of manageable size, aircraft were consolidated into groups. Each group is created from aircraft of the same year group which have similar accumulated flight times. These groups are now treated as a single entity with regards to transference to the USNR. In other words, if one aircraft in a group is transferred to the USNR, the entire group must be transferred. To ensure compliance in the model, the transfer and retirement variables are represented as binary variables, while the update variables are allowed to be general integer variables.

The maximum number of aircraft possible in each group is entered along with the data set as a parameter (data set entry will be discussed in chapter 4) and must be chosen such that it is large enough to reduce the number of variables but small enough to allow flexibility and maintain fidelity. If an excessive number were chosen, it would require the transfer or retirement of a large block of aircraft, which may cause problems in satisfying model constraints. In this model a maximum group size of four or eight was used. The former was chosen to keep the total number of groups at approximately 100, while the latter reduced the problem size and decreased model run times.

A. ASSUMPTIONS

A few assumptions were made in the development of this model. They were made to simplify the model and prevent nonlinearity.

New production aircraft are assumed to remain in the inventory until after the end of this model's planning horizon. Therefore, they cannot be retired. They are also assumed to be technologically able to accomplish their mission until the end of the planning horizon. This assumes they will not undergo an update. As the update 4 incorporates the same technology, this assumes there will be no further updates required on existing aircraft which are modernized by the update 4 upgrade.

Since the planning horizon is to be 20 years or less, the maximum annual flight time accumulation rate is 720 hours, and the maximum flight time ceiling is 20,000 hours, new production aircraft cannot exceed the ceiling during the planning horizon. Thus, flight time is not kept for these aircraft.

As previously mentioned, an additional assumption was made to limit the maximum possible age of an aircraft to 40 years and to limit the maximum group size to four or eight.

To reduce the number of constraints, a production line is required to remain open at least six years. This reduces the number of production line opening and closing year combinations.

B. MODEL CONSTRAINTS

A general description of the model follows. This description is written in a fairly non-technical manner, with a minimum number of equations and notation. For a detailed algebraic description of the model, refer to Appendix A.

1. Balance Constraints.

The options available to a given aircraft group depends on the service to which it is attached. Using one year time increments, USN aircraft can be either updated, transferred, retired or remain in their current status, while USNR aircraft can only be updated, retired or maintain their current status. New production aircraft only have the option of remaining in USN or being transferred to USNR and remaining there until the end of the planning horizon. Inventory balance constraints must be imposed to allow only one option for each individual aircraft. To improve fidelity, the aircraft within a group can be updated in different years, as long as they are transferred or retired as a complete group. As discussed previously, new production aircraft can only be transferred or maintain their current status. The constraints comprise a network flow sub-problem. The inflow for each year is the inventory variable from the previous year, and the outflow is the sum of the aircraft assigned to the various options available in the current year. For instance, the equation for current inventory USN aircraft could be:

$$I_{ga,t-1,r} - U_{gt} - T_{gt} - R_{gt} - I_{gatr} = 0$$

Here, an aircraft, associated with cohort group g and model type a , enters year t assigned to service r ($I_{ga,t-1,r}$). During year t , the aircraft can be either updated (U_{gt}), transferred (T_{gt}), retired (R_{gt}) or retain current model type a and remain assigned to service r (I_{gatr}).

2. Required Inventory Constraints.

Each service (USN, USNR) has its own operational requirements and minimum number of aircraft required to accomplish them. The model uses an elastic constraint which sums over all aircraft cohort groups and model types in each service.

If the service total does not fall between the minimum and maximum desired inventory level, a penalty is assessed in the objective function.

3. High Technology Constraints.

Each aircraft model type is designated as to whether or not it is considered to contain high technology avionics in year t . Since each service type has its own operational commitments, a different "high tech" goal may be required. The model sets the desired level for each service type for each year in the planning horizon. The model then uses an elastic constraint to sum over all aircraft cohort groups and model types, and determine the percentage of aircraft which are considered high tech for each service in a given year. If the percentage does not meet the specified goals for year t ($HMIN_{it}$), a penalty is assessed in the objective function.

Another set of constraints could be added which would designate an intermediary point. Aircraft could be classified as having "medium" technology when the avionics is still very capable but is unable to adequately accomplish a mission against the newest threats. Other variations to this theme could also be added.

4. Mean Age Constraints.

To ensure that the average age of the fleet does not rise above a designated level for each service type, the model uses an elastic constraint to sum the age of all aircraft in a given service type in a given year. The average age of each service type is determined and if it exceeds the desired limit ($AMEAN_{it}$), a penalty is assessed in the objective function.

5. Flight Time Constraints.

Each aircraft has a specified maximum flight time limit. Once an aircraft reaches this limit, it must undergo SDLMs every 40 months. To estimate the future flight time for a given aircraft group, the annual flight time hour accumulation rate for the USN/USNR (FTR_{it}) is used. The inventory variable is multiplied by the appropriate value of FTR and this value is added to the previous year's total (flight time values are initialized for each group in the data entry file). Since a group can only be attached to the USN or USNR and must be retired concurrently, there is no ambiguity if the maximum flight time ceiling is exceeded. If it is exceeded, all aircraft in the group have exceeded it and are penalized the value of initiating mandatory

SDLMs prior to reaching 30 years of age (if the group's age is already at least 30 years, there is no penalty). This penalty equals the number of years remaining until aircraft reaches 30 years of age times one third the mandatory SDLM's cost (each SDLM is valid for 40 months which is approximately 3 years).

6. Budgetary Constraints.

In dealing with Navy operating expenses, budgetary authority is divided into two primary categories. All procurement and upgrade expenses are accounted for in the Aircraft Procurement Navy (APN) budget. The operating and maintenance (O&M) budget handles the routine daily expenses, SDLM, transfer, and retirement costs. Therefore, the model uses two separate constraints to ensure that neither the APN nor O&M budgets are exceeded.

7. Production Constraints.

There are four types of constraints dealing with each production line (new aircraft and update kit), and one which involves both lines. Each line has a physical or economic minimum and maximum annual production capacity. The maximum number usually corresponds to actual physical limitations of production. The minimum number usually reflects the number required to be produced to effectively employ the workforce. Therefore two sets of equations ensure the bounds are met. The minimum bound is elastic since it typically involves economic considerations and for a hefty penalty can be overcome. However, since the upper bound is usually caused by physical considerations, it is not elastic and must not be exceeded. Since other countries purchase the aircraft and update kits, the expected Foreign Military Sales (FMS) are included in the above constraints.

Another aspect of many purchasing agreements is the minimum purchase obligation. The agency agrees to purchase at least a specified number of units within a certain timeframe of the contract. So, in addition to the minimum and maximum annual procurement constraints, an elastic cumulative (from the initial year of the contract) constraint ensures the number of units purchased by the y^{th} year of a contract satisfies all obligations.

In the model, a binary indicator variable is designated for each possible combination of opening and closing years for a given line. The actual opening and

closing years of the line determine the "production campaign" for that particular line. The variables play an integral part in the previous three constraints. Each of the constraints are dependant on the number of years since the beginning of the campaign. For instance, a workforce may experience a learning curve on a new production line and require a slower production rate during the first few years of a production campaign. The maximum limit in this case would be smaller for the earlier years of the campaign.

The final constraint for each production line, is to ensure the solver selects only one production campaign from the numerous possibilities. This unique production campaign will drive the previous three production line constraints.

Since the new aircraft production line is dependant on the update kit line (because it uses the same kits), a constraint is necessary to ensure that the new aircraft production campaign is a subset of the update kit line's "time frame". That is, the update kit line must open at the same time or prior to, and close concurrently with or after the new aircraft production line.

8. Objective Function.

The objective function of the model is to minimize APN and O&M costs, taking into account the various penalties associated with the elastic constraints.

IV. IMPLEMENTATION

Once the model is devised, a suitable solver must be found to implement it. Due to the potential size of this model (9000+ constraints by 20000+ variables), a large-scale solver must be invoked. The X-system solver [Ref. 5] was chosen since it has been used successfully to solve models of this magnitude and it was used to solve the Phoenix model. The X-system uses a sparse matrix format which reduces the storage requirements for the constraint matrix. Therefore, the data must be organized into various matrices and arrays to be fed into the solver. The X-System's output is an array containing the values for each of the model constraints and variables. This output can then be used to create a summary report.

A. X-SYSTEM PREPROCESSING

To use the X-System, a binary data file must be provided, which contains all the necessary arrays. This binary file is established by identifying all of the non-zero coefficients and their locations in the model constraint matrix. This would be extremely difficult and time consuming to create manually. To aid in this process, three items must be created by the modeler and user. A data input file is used to transfer raw data from the user to the modeler's programs. The raw data contains the pertinent values which are used to calculate the coefficients utilized in the arrays required by the matrix generator described below. An editor program is used to transform this raw data into completely specified arrays. A matrix generator program uses the arrays created by the editor, to create a Linear Programming (LP) constraint matrix. This matrix is stored using linked lists to conserve storage space, and sent to the X-System solver.

1. DATA INPUT FILE

To make the model responsive and flexible, an input data file is used to insert the various raw data parameters. The file is a "fill-in-the-blank" template which is completed by the user prior to each run. The concept behind the input data file is to avoid complete enumeration of the parameter matrices required by the model.

Instead, pertinent information is placed in the input data file, and read by a subroutine (EDITOR) which creates and stores the complete matrices required. A series of "what-if" situations can be created by changing a parameter in one place (the data input file), and the EDITOR program will complete all subsequent modifications. Additionally, by only changing the data input file, the model can be run without having to change and recompile any of the FORTRAN subroutines.

The structure of the input data file and a description of the various components, as well as the data input file implemented for this report, is contained in Appendix B.

2. EDITOR PROGRAM

The EDITOR subroutine takes the raw data parameters and creates the arrays required by the matrix generator. The input data file may provide a series of values for one index of an array which are identical or related algebraically to others in the same (or other) array. The EDITOR programs completes the necessary manipulations and completely specifies the array and stores it for use by the matrix generator subroutine (MATGEN).

3. MATRIX GENERATOR (MATGEN)

The matrix generator uses the arrays created by the EDITOR subroutine to create the LP constraint matrix. Since complete enumeration of the constraint matrix would be impractical due to its size (and subsequent CPU storage requirements), and to utilize the X-System, the constraint matrix is stored in a sparse format. The matrix generator program creates a compressed version of the constraint matrix and objective function using linked lists [Ref. 6]. Using the specified model, constraints are generated and variables created in order to establish the row and column identifications for the constraint matrix. The generator then proceeds to identify each non-zero coefficient within this matrix framework. The location and value for the coefficients are then stored in the linked lists. The matrix generator then writes all of the linked list arrays, which define the constraint matrix, to a binary data file. The binary data file is then used by the X-System to solve the model.

B. X-SYSTEM POST-PROCESSING

1. REPORT WRITER

The output from the X-System solver is an array which contains the value for each row (constraint) and column (variable) in the constraint matrix. To make this data appear in a coherent and operationally useful manner, a report writer program is created. The report writer manipulates the output data and creates tables which depict the information required by the user. The output from the report writer used for this model is contained in Appendix D.

C. X-SYSTEM IMPLEMENTATION

Validation of the model and pre-processing subroutines was accomplished by downscaling the problem to a few aircraft groups and a two year planning horizon. Once validated, the full scale problem was run for time horizons of five to ten years. The X-System experienced numerical difficulties in solving the LP relaxation when the time horizon exceeded five years. Even when the LP relaxation could be solved, problems with the ILP solver prevented an optimal integer solution from being found.

The numerical difficulties encountered in performing pivots has not been solved to date. Scaling the objective function coefficients was marginally effective for this problem and allowed the model to solve faster. Through other software modifications, the LP relaxation was successfully solved for a time horizon of eight years. A full 20 year planning horizon model run was not possible due to these numerical problems.

Since the ILP solution algorithm used in the X-System was unable to solve the model optimally (regardless of the outcome of the LP relaxation), a heuristic algorithm was written to provide a solution (HILP). The heuristic solution is not guaranteed to be optimal but it should be fairly close.

1. HEURISTIC INTEGER ROUNDING ALGORITHM

The heuristic algorithm takes the LP solution, fixes some variables at a certain value, and then runs the LP solver again. The heuristic initially finds the new aircraft production campaign which covers the span created by any production campaign which was opened in the LP solution. The algorithm then sequences through each year of the planning horizon and sets variables in the following order: original inventory group transfers (T), new aircraft production (P), new production.

aircraft group transfers (T), and retirements (R). Following each set of variables being fixed, the LP solver is run. If the LP and the rounding algorithm are identical for a set of variables, the LP solver is not run again, and the next set of variables is fixed. After fixing the above variables for every planning horizon year, the update variables (U) are rounded to provide an integer solution to the model.

The heuristic algorithm could require up to $\{2 + (4 * (\# \text{ Planning Horizon years}))\}$ runs of the LP solver. As the number of planning years increases, the length of time required for a single LP run increases at a superlinear rate. This is a serious problem which must be resolved if this model is to be used in real world "what if" scenarios.

D. COMPUTATIONAL EXPERIENCE

In addition to the numerical difficulties mentioned above, two other areas of concern were identified. The model requires a large amount of RAM (random access memory) and takes a long time to solve (TABLE 1). As previously mentioned, increasing the objective function scale factor can decrease the CPU processing time by 20-30%. A level is reached though, where further increases prove counter-productive. The values depicted in Table 1, reflect the best scaling method observed.

In addition to detailing the storage and time requirements for various time horizon lengths, Table 1 provides the LP and HILP solutions to the model. In the five year time horizon, the HILP differs from the LP by 3-5%. This is perfectly reasonable, given the LP will fractionate variables and run multiple production lines simultaneously. These multiple production lines must be coalesced into one, resulting in higher costs. Additionally, the LP solution allows for fractions of cohort groups to be retired or transferred, while the HILP requires the entire cohort group to be retired or transferred. The large percentage difference between the HILP and LP objective solutions for the eight year planning horizon is caused by multiple new aircraft production lines being opened. After these lines are coalesced into a single production campaign, the LP objective function value is 11.82, which represents a 3.1% difference from the HILP solution. As with the five year time horizon, this is caused by the fractionation of cohort groups and production amounts.

TABLE 1. MPA MODEL RUN STATISTICS

# YEARS IN PLANNING HORIZON	5	5	8
MAX GROUP SIZE	4	8	4
# CONSTRAINTS	2340	1363	3827
# VARIABLES	5368	3376	9551
MEMORY REQUIRED (Megabytes)	2.3	1.5	3.0
LP CPU TIME (SEC)	261.3	117.5	1031.3
HILP CPU TIME	1868.6	880.5	10927.1
LP OBJ FUNCTION SOLN	7.112	7.112	10.90
HILP OBJ FUNCTION SOLN	7.383	7.465	12.19
% DIFFERENCE BTWN LP & HILP	3.8%	4.8%	11.8

By increasing the maximum group size, the number of variables and constraints decrease, as does the CPU time (Table 1). This is achieved at the expense of model accuracy. Increasing the maximum group size will cause greater fluctuation from the minimum inventory level. This is caused by the HILP rounding algorithm which fixes the retirement and transfer variables to one when the variable value from the LP solver is greater than a set value (currently 0.7 for transfer and 0.80 for retirement). This is illustrated for a time horizon of five years and maximum group sizes of four and eight (Figure 3).

Maximum Group Size = 4

YEAR	TOTAL INV	USNR INVENTORY LEVELS		BMOD	AIRCRAFT MODEL TYPES				P7
		DESIRED MIN	MAX		CU1	CU2	CU3	CU4	
1991	212.0	212	274	0.0	34.0	65.0	107.0	6.0	0.0
1992	202.0	202	274	0.0	21.0	65.0	98.0	19.0	0.0
1993	202.0	202	274	0.0	6.0	61.0	80.0	56.0	0.0
1994	203.0	202	274	0.0	0.0	50.0	56.0	97.0	0.0
1995	202.0	202	274	0.0	0.0	39.0	26.0	134.0	3.0

YEAR	TOTAL INV	USNR INVENTORY LEVELS		BMOD	AIRCRAFT MODEL TYPES				P7
		DESIRED MIN	MAX		CU1	CU2	CU3	CU4	
1991	77.0	75	96	41.0	0.0	4.0	26.0	4.0	0.0
1992	73.0	70	96	33.0	3.0	4.0	25.0	8.0	0.0
1993	70.0	70	96	29.0	3.0	4.0	25.0	8.0	2.0
1994	70.0	70	96	29.0	1.0	4.0	22.0	13.0	4.0
1995	73.0	70	96	21.0	1.0	8.0	15.0	20.0	6.0

Maximum Group Size = 8

YEAR	TOTAL INV	USNR INVENTORY LEVELS		BMOD	AIRCRAFT MODEL TYPES				P7
		DESIRED MIN	MAX		CU1	CU2	CU3	CU4	
1991	216.0	216	274	0.0	34.0	66.0	116.0	0.0	0.0
1992	204.0	208	274	0.0	24.0	66.0	105.0	13.0	0.0
1993	204.0	202	274	0.0	11.0	60.0	70.0	61.0	0.0
1994	204.0	204	274	0.0	0.0	50.0	50.0	97.0	2.0
1995	202.0	202	274	0.0	0.0	42.0	42.0	130.0	7.0

YEAR	TOTAL INV	USNR INVENTORY LEVELS		BMOD	AIRCRAFT MODEL TYPES				P7
		DESIRED MIN	MAX		CU1	CU2	CU3	CU4	
1991	77.0	75	96	53.0	0.0	1.0	18.0	5.0	0.0
1992	69.0	70	96	41.0	8.0	1.0	18.0	5.0	0.0
1993	71.0	70	96	33.0	8.0	1.0	24.0	5.0	1.0
1994	73.0	70	96	33.0	5.0	1.0	18.0	14.0	2.0
1995	76.0	70	96	25.0	2.0	1.0	18.0	24.0	3.0

Figure 3. Partial Summary Report for Five Year Time Horizon

V. CONCLUSION AND RECOMMENDATIONS

A. CONCLUSIONS

The Maritime Patrol Aircraft modernization model is computationally, very difficult. Due to the computational difficulties, a full 20 year time horizon run was not possible. However, the runs performed using lesser time horizons demonstrated the utility of the model. The user is presented with a blueprint as to when to retire, transfer, or update existing fleet aircraft, as well as when to open a production line and the number of aircraft to procure each year.

Once the problems with the X-System solver are resolved though, this model offers an extremely flexible method to determine a modernization schedule for the MPA fleet. The model is easily modified to facilitate different priorities or concerns. The ability to change parameters in the data input file makes the model very conducive to "what-if" scenario analysis.

B. OTHER POTENTIAL USES

The fleet modernization problem is not unique to the MPA fleet. The problem is being experienced by almost every aircraft community in the Navy. Since each community is relatively small with respect to numbers of aircraft, integer solutions to any model would be required. The operational and budgetary limitations could be very similar to those encountered in the MPA model. If this is the case, the MPA model could be easily modified to reflect any differences. The model is structured such that these differences can be fairly significant, and yet require only slight modifications to the underlying FORTRAN code. Therefore, the model is applicable not only to the MPA community but to any small (in terms of numbers of units) community which requires a modernization program. Additionally, communities with large numbers of aircraft could use a continuous inventory variable version. For large numbers of aircraft, it would be significantly important to have an integer answer. A value of 150.5 is adequate when 151 actually are actually required.

C. RECOMMENDATIONS

To make the model more effective, a solution to the run time requirements must be found. The computational problems experienced by the X-System solver must be reduced and the integer solver problem must be resolved. If this is not possible, or if the run time requirements are still excessive, another solver may have to be used.

An additional area of potential study, is to investigate decomposing the model into two sub-problems, each dealing with one service. Since the majority of computational difficulty is experienced when the transfer of assets is included into the model, decomposition may reduce these difficulties.

APPENDIX A. MPA UPGRADE MODEL ALGEBRAIC DESCRIPTION

A. INTRODUCTION.

The MPA procurement model is a mixed integer program. The following provides the basic mathematical description of the model, as well as a brief description of the components. Appendix B provides more detailed summary of the input parameters and rationale for them.

B. INDEX USAGE.

1. Indices:

- (a) a Aircraft model type
- (b) g Aircraft cohort year group
- (c) t Planning (fiscal) year
- (d) r Branch of service to which aircraft assigned
- (e) y y^{th} year of production campaign
- (f) v Start year of production campaign
- (g) w End year of production campaign
- (h) k Indicates number of elements in set (i.e., number of aircraft produced in given year)
- (i) c Indicates aircraft's age.

2. Basic Index Sets:

- (a) A Aircraft model types a
- (b) A' $A' \subseteq A$; Current aircraft inventory model types a .
- (c) A'' $A'' \subseteq A$; Aircraft update model types.
- (d) A''' $A''' \subseteq A$; Aircraft new production model types.
- (e) A'''' $A'''' \subseteq A'$; Aircraft models which can be updated to model types in A'' .

- (f) A_g Possible model types for cohort group g . This includes the original model type and possible update model types, for each cohort group.
- (g) G Aircraft cohort year groups g .
- (h) G' $G' \subseteq G$; Current inventory cohort groups g .
- (i) G'' $G'' \subseteq G$; New production aircraft cohort groups g .
- (j) T Planning (fiscal) years t .
- (k) R Branch of service ($r' = \text{USN}$, $r'' = \text{USNR}$).
- (l) Y Possible production campaign years y . Numbering starts from zero and corresponds to the first year of a production campaign.
- (m) V Possible start years v for production campaign.
- (n) W Possible end years w for production campaign.
- (o) VWP_{vw} Possible combinations of new aircraft production campaign start year $v \in V$ and end year $w \in W$ such that $v < w$ and minimum line open times are met.
- (p) VWU_{vw} Possible combinations of update kit production campaign start year $v \in V$ and end year $w \in W$ such that $v < w$ and minimum line open times are met.
- (q) $TVWP_{tvw}$ A value of "1" indicates that the new production line could be open in year t , given a campaign start year $v \in V$ and end year $w \in W$ ($v \leq t \leq w$).
- (r) $TVWU_{tvw}$ A value of "1" indicates that the update kit production line could be open in year t , given a campaign start year $v \in V$ and end year $w \in W$ ($v \leq t \leq w$).
- (s) K Possible numbers k . Used in conjunction with new aircraft production. Maximum number is the largest possible new aircraft production line operating level for any year in planning horizon.
- (t) C Possible ages for any aircraft in fleet.

C. MODEL DATA

1. Cost and Technology Data

- (a) CP_{ak} Cost of producing one aircraft of model type aeA'' when k aircraft are purchased in a given year. This only includes recurring unit costs.
- (b) CU Cost of updating aircraft to model type A'' (This value is constant for all aircraft cohort groups except BMODs which are not allowed to be updated in model). This only includes recurring unit costs.
- (c) CFP_{tvw} Fixed costs associated with operating new aircraft production line in year t , given production begins in year v and ends in year w .
- (d) CFU_{tvw} Fixed costs associated with operating update kit line in year t , given update kit production begins in year v and ends in year w .
- (e) CS_c Standard Depot Level Maintenance (SDLM) costs for aircraft of age c years.
- (f) CO_{grt} Annual operating and maintenance costs to operate aircraft in cohort group g , assigned to service type r , in year t .
- (g) CR Cost of retiring one aircraft.
- (h) CT Cost of transferring one aircraft to USNR.
- (i) $IMIN_{tr}$ Minimum inventory level, of all aircraft model types combined, required by service type r , in year t .
- (j) $IMAX_{tr}$ Maximum inventory level, of all aircraft model types combined, required by service type r , in year t .
- (k) H_{at} Binary digit has a value of "1" if aircraft model type a is considered to have high technology avionics in year t , and "0" otherwise.
- (l) $HMIN_{tr}$ Minimum percentage of total aircraft inventory containing high technology avionics required by each service type r in year t .

- (m) $AMEAN_{tr}$ Maximum mean age of all aircraft in the inventory, acceptable to service type r in year t .
- (n) $AMAX_g$ Maximum age allowed for aircraft cohort group g .
- (o) FTR_r Flight time rate for service type r .
- (p) $PMIN_y$ Minimum number of new aircraft required to be produced in the y^{th} year of contract.
- (q) $PMAX_y$ Maximum number of new aircraft which can be produced in y^{th} year of contract.
- (r) $PMINREQ_y$ Minimum number of new aircraft required to be produced by the y^{th} year, of a production campaign, in order to meet current contractual requirements.
- (s) $UMIN_y$ Minimum number of aircraft update kits required to be produced in the y^{th} year of a contract.
- (t) $UMAX_y$ Maximum number of aircraft update kits which can be produced in y^{th} year of contract.
- (u) $UMINREQ_y$ Minimum number of update kits required to be produced by the y^{th} year of an update campaign, in order to meet current contractual requirements.
- (v) $BMAXP_t$ Maximum budget level, for new aircraft purchases (Aircraft Procurement Navy 1 funds (APN1)) plus aircraft retrofits (APN5 funds), anticipated in fiscal year t .
- (w) $BMAXO_t$ Maximum budget level for operating and maintenance, SDLM, and aircraft retirements, anticipated in fiscal year t .
- (x) LAG_g Lag time between fiscal allocation and delivery of new aircraft for aircraft cohort group g . Aircraft are given a cohort group reflecting the procurement, not delivery year.
- (y) $FT0_g$ Flight time accrued by current inventory cohort year group $g \in G'$ at model start time.
- (z) FMS_t Number of Foreign Military Sales (FMS) of new aircraft expected in year t .

- (aa) $FTMAX_g$ The maximum flight time an aircraft in cohort group $g \in G'$ can fly without undergoing mandatory SDLM every 40 months.
- (bb) AGE_{gt} The age of aircraft associated with cohort group g , in year t . This age reflects the time since delivery to the fleet and therefore is $t - (\text{cohort year}) + LAG_g$.

2. Penalty Weights

- (a) PAI_{tr} Penalty assessed per unit deviation above upper limit inventory levels set by $IMAX_{tr}$ for service type r , in year t .
- (b) PBI_{tr} Penalty assessed per unit deviation below lower limit inventory levels set by $IMIN_{tr}$ for service type r , in year t .
- (c) PBH_{tr} Penalty assessed per unit deviation of the high technology lower limit set by $HMIN_{tr}$ for service type r , in year t .
- (d) PAA_{tr} Penalty assessed per unit deviation above the mean age upper limit set by $AMEAN_{tr}$ for service type r , in year t .
- (e) $PABP_t$ Penalty assessed per unit deviation above upper APN1 and APN5 total budgetary limit set by $BMAXP_t$ for year t .
- (f) $PABO_t$ Penalty assessed per unit deviation above upper operating and maintenance budgetary limit set by $BMAXO_t$ for year t .
- (g) PPB Penalty assessed per unit deviation below minimum cumulative production levels set forth in contractual obligations specified by $PMINREQ_y$.
- (h) PBP_t Penalty assessed per unit deviation below minimum new aircraft production levels set by $PMIN_y$. Penalty value is fixed for a given year t , regardless of the number of years since the start of a production campaign.
- (i) PUB Penalty assessed per unit deviation below minimum cumulative update kit production levels set forth in contractual obligations specified by $UMINREQ_y$.

- (j) PBU_t Penalty assessed per unit deviation below minimum update kit production levels set by $UMIN_t$ for year t . Penalty value is fixed for a given year t , regardless of the number of years since the start of a production campaign.

D. DECISION VARIABLES

1. I_{gatk} Number of aircraft of type a and associated with cohort year group g which are contained in the inventory of service type r in year t . For current inventory cohort groups ($g \in G'$), $I_{gatr} \in \{0, NUM_g\}$. The k subscript is not used for original inventory aircraft since I is a general integer variable and is set to "1".
For new production aircraft, the variable I is binary and the index k refers to the number of aircraft associated with cohort group g , of type aeA and service type r .
2. T_{gt} Binary variable has a value of "1" if all aircraft associated with cohort group g are transferred to the reserves in year t , "0" otherwise.
3. R_{gt} Binary variable has a value of "1" if all aircraft associated with cohort year group g are retired in year t , "0" otherwise.
4. P_{atk} Binary variable which has a value of "1" when k aircraft, of type aeA are produced in year t , "0" otherwise.
5. U_{gt} Number of aircraft associated with cohort year group g which are updated in year t . For current inventory cohort groups ($g \in G'$), $U_{gt} \in \{0, NUM_g\}$.
6. OP_{vw} Binary variable has a value of "1" when new aircraft production campaign begins in year v and ends in year w , "0" otherwise.
7. OU_{vw} Binary variable has a value of "1" when aircraft update kit campaign begins in year v and ends in year w , "0" otherwise.

E. CONSTRAINT VIOLATION VARIABLES

1. AI_{tr} The number of aircraft above the desired maximum inventory level, specified by $IMAX_{tr}$, for service type r , in year t .

2. BL_{tr} The number of aircraft below the desired minimum inventory level, specified by $IMIN_{tr}$, for service type r , in year t .
3. BH_{tr} The number of additional high technology aircraft required to meet minimum high technology level, specified by $HMIN_{tr}$, for service type r in year t .
4. AA_{tr} The number of aircraft years above desired maximum mean aircraft age level, specified by $AMEAN_{tr}$, for service type r , in year t .
5. ABP_t The amount expenditures exceed APN1 and APN5 total maximum budgetary level set by $BMAXP_t$ for year t .
6. ABO_t Amount expenditures exceed maximum operating and maintenance budgetary level set by $BMAXO_t$ for year t .
7. PB_t The number of aircraft in violation of minimum required to meet contractual requirements, specified by $PMINREQ_t$, in year t .
8. UB_t The number of update kits in violation of minimum required to meet contractual requirements, specified by $UMINREQ_t$, in year t .
9. BP_t Number of new production aircraft short of minimum required production levels set by $PMIN_t$ for year t .
10. BU_t Number of aircraft update kits short of minimum required production levels set by $UMIN_t$ for year t .
11. OF_{gt} Binary variable has a value of "1" when flight time limit was violated by cohort year group g in year t , "0" otherwise.

F. CONSTRAINTS

1. USN Aircraft Inventory Balance: $te\{T \mid 0 \leq AGE_{gt} \leq AMAX_g\}$, aeA , geG , $r=r'$

During year t , aircraft associated with cohort year group g (and starting year in USN) can be either retired (R), transferred to the reserves (T), or updated from model type aeA''' (U) to model type aeA'' . New production aircraft may only be transferred to the reserves. Since "R" and "T" are binary variables, they must be multiplied by the number of aircraft in the cohort group.

geG', aeA' :

$$\sum_k (k * I_{ga,t-1,rk}) - U_{gt} - (T_{gt} * NUM_g) - (R_{gt} * NUM_g) - \sum_k (k * I_{gark}) = 0$$

geG', aeA'' :

$$\sum_k (k * I_{ga,t-1,rk}) + U_{gt} - (T_{gt} * NUM_g) - (R_{gt} * NUM_g) - \sum_k (k * I_{gark}) = 0$$

geG'', aeA''' :

$$\sum_k (k * I_{ga,t-1,rk}) + (k * P_{akt}) - (T_{gt} * NUM_g) - \sum_k (k * I_{gark}) = 0$$

2. USNR Aircraft Inventory Balance: $te\{T \mid 0 \leq AGE_{gt} \leq AMAX_g\}$, aeA , geG , $r=r''$

During year t , aircraft associated with cohort year group g (and starting year in USNR) can be either retired (R), or updated from model type aeA' (U) to model type aeA'' . New production aircraft may not be retired or updated.

geG', aeA' :

$$\sum_k (k * I_{ga,t-1,rk}) + (T_{gt} * NUM_g) - U_{gt} - (R_{gt} * NUM_g) - \sum_k (k * I_{gark}) = 0$$

geG', aeA'' :

$$\sum_k (k * I_{ga,t-1,rk}) + U_{gt} + (T_{gt} * NUM_g) - (R_{gt} * NUM_g) - \sum_k (k * I_{gark}) = 0$$

$g \in G, a \in A$:

$$\sum_k (k * I_{g,a,t-1,r,k}) + (k * P_{ark}) + (T_{gt} * NUM_g) - \sum_k (k * I_{gark}) = 0$$

3. Aircraft Inventory Levels: $t \in T, r \in R$

Elastic constraint which bounds the total aircraft inventory required by each service type r , in year t .

$$IMIN_r - BI_r \leq \sum_a \sum_g \sum_k k * I_{gark} \leq IMAX_r + AI_r$$

4. High Technology Requirement: $t \in T, r \in R$

Elastic constraint which sets a desired lower limit on the percentage of aircraft satisfying the high technology definition.

$$- BH_r \leq \sum_a \sum_g \sum_k (H_{at} - HMIN_r) * k * I_{gark}$$

5. Mean Age Requirements: $t \in T, r \in R$

Elastic constraint which sets a desired upper limit on the mean age of aircraft in each service type r , in year t . The maximum allowable age requirement will be addressed through index set manipulation.

$$\sum_g \sum_{a \in A_g} \sum_k (AGE_{gt} - AMEAN_r) * k * I_{gark} \leq AA_r$$

6. Flight Time Requirements: $t \in T, g \in G'$

Updates the flight time for each aircraft cohort group g , using the flight time rate for the service type it is attached to in year t . The value of M is an arbitrarily large constant (however, as small in magnitude as possible). (Note: $k=1$ for all inventory variables in these constraints)

$$\sum_{t' < t} \sum_a \sum_r (I_{g,at'rk} * FTR_r) - M * OF_{gt} \leq NUM_g * (FTMAX_g - FT0_g)$$

7. New Aircraft Production Line: $a \in A''$

Production Line must be opened and closed.

$$\sum_{(v,w) \in TVWP} OP_{avw} = 1$$

8. New Aircraft Production Line Rate: $(t,v,w) \in TVWP_{tvw}, a \in A''$

Enforces a lower and upper limit on number aircraft produced on line in year t .

$$FMS_t - BP_t \leq \sum_r (K * P_{atrk}) - \sum_{(v,w) \in TVWP} (OP_{avw} * PMIN_{t-v})$$

$$\sum_{(v,w) \in TVWP} (OP_{avw} * PMAX_{t-v}) - \sum_r (k * P_{atrk}) \leq FMS_t$$

9. Cumulative New Aircraft Production Quantity Requirement: $(t,v,w) \in TVWP_{tvw}, a \in A''$

Elastic constraint which makes it advantageous (i.e. to meet contractual requirements) to produce specified quantities by year t when production campaign is started in year v .

$$\sum_{(v,w) \in TVWP} (OP_{avw} * PCUMREQ_{t-v}) - \sum_{t' < t} \sum_r (k * P_{atrk}) \leq PB_t$$

10. Update Kit Production Line: $a \in A''$

Update Line must be opened and closed.

$$\sum_{(v,w) \in VWU} OU_{avw} = 1$$

11. Update Kit Production Line Uniqueness: $a' \in A''$, $a'' \in A'''$, $(v,w) \in VWP_{vw}$

Ensures production line is open when aircraft update kits are produced. An update line must be open when kits are produced for either retrofits or new production.

$$OP_{a'''vw} - \sum_{(v,w) \in VWU | (v' \leq v, w' \leq w)} OU_{a''v'w'} \leq 0$$

12. Update Kit Line Production Rate: $a' \in A''$, $a'' \in A'''$, $(t,v,w) \in TVWU_{tvw}$

Elastic equation which enforces a lower and upper limit on number aircraft updated on line in year t.

$$-BU_t \leq \sum_g \sum_r \sum_{a \in A''''} U_{gar} + \sum_k (k * P_{a'''sk}) - \sum_{(v,w) \in VWU} (OU_{a''vw} * UMIN_{t-v})$$

$$\sum_{(v,w) \in VWU} (OU_{a''vw} * UMIN_{t-v}) - \sum_k (k * P_{a'''sk}) - \sum_g \sum_r U_{gar} \leq 0$$

13. Cumulative Update Kit Quantity Requirement: $a \in A''''$, $a' \in A''$, $a'' \in A'''$, $(t,v,w) \in TVWU_{tvw}$

Elastic constraint which makes it advantageous (i.e. to meet contractual requirements) to produce specified quantities by year t when production campaign is started in year v.

$$\sum_{(v,w) \in VWU} (OU_{a''vw} * UCUMREQ_{t-v}) - \sum_g \sum_a \sum_{t' \leq t} \sum_r U_{gar'} + \sum_{t' \leq t} \sum_k (k * P_{sk}) \leq UB_t$$

14. Budgetary Requirements: teT

An elastic maximum budgetary limit is imposed on MPA operations. The budget consists of 3 primary areas, each with their own monetary allowance (APN1, APN5, Operating and Maintenance (O&M)). The budgetary constraint is separated into two equations (one for APN1/5; one for O&M). The component costs include (corresponding to each line of equation):

APN1 & APN5 considered jointly:

- variable per unit costs dependant on number new aircraft procured (including update kit)
- fixed new aircraft production line costs
- penalty costs for breaking new aircraft contractual production requirements
- variable per unit avionics update kit costs
- fixed update kit line costs
- penalty costs for breaking update kit contractual production requirements

$$\begin{aligned} & \sum_{a \in A'''} \sum_k (k * CP_{ak} * P_{atk}) + \sum_{a \in A'''} \sum_{(v,w) \in VWP} (CFP_{vw} * OP_{avw}) \\ & + \sum_g \sum_r (CU * U_{gr}) + \sum_{a \in A''} \sum_{(v,w) \in VWU} (CFU_{vw} * OU_{avw}) \\ & + (BU_t * PBU_t) \leq BMAXP_t + ABP_t \end{aligned}$$

Operating and Maintenance:

- SDLM costs depending on age of aircraft
- SDLM costs for aircraft exceeding flight time ceiling
- Operating and Maintenance costs for aircraft
- Retirement costs
- Transfer costs

$$\begin{aligned} & \sum_g \sum_a \sum_r \sum_k (CS_t * k * I_{gatr}) + \sum_{g | AGE_g < 30} (1/3 * CS_{30} * OF_g) \\ & + \sum_g \sum_a \sum_r (CO_{gr} * I_{gatr}) + \sum_g (CR * R_g) \\ & + \sum_g (CT * T_g) \leq BMAXO_t + ABO_t \end{aligned}$$

G. OBJECTIVE FUNCTION

Minimize total cost and elastic variable penalties.

Minimize

$$\sum_i \left[\begin{aligned} & \sum_{aaA'''} \sum_k (k * CP_{ak} * P_{ask}) + \sum_{aaA'''} \sum_{(v,w) \in VWTP} (CFP_{vw} * OP_{avw}) \\ & + \sum_g \sum_r (CU * U_{gr}) + \sum_{aaA''} \sum_{(v,w) \in VWU} (CFU_{TVW} * OU_{avw}) \\ & + \sum_g \sum_a \sum_r \sum_k (CS_i * k * I_{gatr}) + \sum_{g | AGE_g < 30} (1/3 * CS_{30} * OF_{gt}) \\ & + \sum_g \sum_a \sum_r (CO_{gr} * I_{gatr}) + \sum_g (CR * R_{gt}) + \sum_g (CT * T_{gt}) \\ & + \text{PENALTIES} \end{aligned} \right]$$

$$\sum_i \left[\begin{aligned} & \sum_r \left[\begin{aligned} & (PAI_r * AI_r) + (PBI_r * BI_r) \\ & + (PBH_r * BH_r) + (PAA_r * AA_r) \end{aligned} \right] \\ & + (PABP_i * AB_i) + (PABP_i * AB_i) + (PAP_i * AP_i) \\ & + (PBU_i * BU_i) + (PAU_i * AU_i) \\ & + (PPB * PB) + (PUB * UB) \end{aligned} \right]$$

APPENDIX B. REQUIRED INPUT DATA SETS

The data sets mentioned in this appendix are required to run the program. The data input file format found in section B-5, is used by a separate Editor program (discussed in Chapter 4) to generate the required data sets. The actual data input file is included for reference, as well as the sources from which they were obtained.

A. COST DATA

The cost data is the driving factor in the determination of an "optimal" solution. The concept behind the different cost parameters is to separate the per unit variable costs and any economy of scale discounts from the fixed production line/start-up costs, while maintaining a realistic view of the contractual pricing/procurement obligations and the associated penalties for breaking those contracts.

1. CP_{ak} : The cost of producing an aircraft of type a , when a given quantity k are purchased in any given year. This cost does not vary from year to year and can take into account some of the production line efficiencies when high volume levels are produced in a given year (economies of scale). To take into account all of the savings accruing from economy of scale purchasing and multi-year procurement, the model would become non-linear. To avoid this, only the per unit purchase price is adjusted for a given number of aircraft purchased in one year. The price can be varied to allow per-unit costs to decrease as the number of aircraft purchased increases for a given year. This will not take into account the cumulative savings generated by multi-year procurement. The military generally deals only in single-year procurement due to the defense budgetary process, so this is not totally unrealistic.

Since the P-7 aircraft is being produced in two stages (airframe and avionics) from two different contractors, the CP coefficient should include the costs for both stages. The CP cost coefficients can be obtained directly from production bids submitted by the producing corporations.

2. CFP_{tw} : The fixed cost estimates for the new aircraft production line are a conglomerate of many factors. Any cost which is incurred as a result of producing any

aircraft, excluding cost of the aircraft itself, is lumped into this parameter. This includes pre-production costs (such as R&D, tooling procurement, etc.) and line start-up/shut-down costs. Since the non-recurring expenses will vary depending on when the production line is started, the indices of the parameter are structured to have the coefficients reflect the costs of the production line in a given year t , when the line is started in year v and stopped in year w . This allows tailoring the matrix to reflect non-recurring costs that would occur in a given year if different decision paths are chosen. If some costs are already sunk under an assumption the line will start in year v , but would have to be reinvested if the line is started in year v' , the former would be more advantageous from a CFP point of view. Additionally, the CFP matrix facilitates amortization of start-up costs over a specified period by adding this amount to the normal fixed costs. In this model a period of six years is used to amortize the start-up costs. The data input would include variable amounts for production campaign years from -6 (to allow for pre-production costs) to +6, with a fixed amount to be applied to each subsequent year until line termination, and then up to 3 years worth of post-production shutdown costs. More thorough research is required into contractor specific cost factors and other potential costs incurred by a decision to start a line in a given year. Some subjective decisions must be made.

The CFP data should NOT include the fixed costs associated with the avionics update kit line. This will be handled by the CFU coefficient.

3. CU: Since the cost to retrofit any "Charlie" model type to an Update IV model type is essentially the same, a constant upgrade cost was used. As with the CP coefficient, the CU coefficient should include only the variable costs of producing an update kit. All fixed costs will be addressed in the CFU coefficient.

There is currently no intention to update the P-3 BMOD aircraft, due to excessive costs for a feasibility study to determine whether the "B" airframe can be modified to a "C" configuration. Additionally, the "B" airframes are approaching the end of their service lives.

In future versions of the model, the CU cost could be easily changed to reflect upgrade costs by cohort group, model type being retrofitted from, or aircraft age. Currently, the fixed upgrade cost is modified within the EDITOR and MATGEN

subroutines to increment the cost by a factor of $(0.01 * \text{sequential group number})$; the sequential group number is the index from the top of the original inventory cohort group data input). This factor will accomplish two things. First, it will make it advantageous (all other factors being equal) to upgrade older aircraft first, and second it will eliminate some of the redundancy in the final solution. The latter factor will decrease the time it takes the solver to find an optimal solution since it will not have to cycle through redundant solutions to each extreme point.

4. $CFU_{a,t,v,w}$: This data is essentially the same as the CFP data except it is associated with the update kit line versus the new aircraft production line. The fixed costs, associated with the retrofit and initial production aircraft avionics kits, are combined into a single factor.

5. CS_y : The cost of SDLM will vary according to the age of the aircraft. Following each SDLM, the aircraft is allowed to continue to operate until the next SDLM cycle. This can conceivably continue forever. However, an arbitrary decision was made to retire an aircraft after it reaches 40 years of age. Prior to being sent to SDLM, an aircraft undergoes an Aircraft Service Period Adjustment (ASPA) inspection which can defer a SDLM for 12 months. If an aircraft continually passes ASPA inspections, it can essentially defer a SDLM forever.

To compute the CS coefficient, the ASPA deferral rate was used, in conjunction with the SDLM costs, to establish an expected value cost for an aircraft of age y . The FORTRAN program SCOST (Appendix C) will output an expected SDLM cost by age of an aircraft (from 0-30). The input is the deferral rate matrix. The matrix consists of the deferral rate for the first through sixth ASPA for the first through seventh SDLM cycles. It is assumed the aircraft will fail an ASPA on its seventh attempt in a SDLM cycle and therefore undergo a SDLM. As data is not accurately available to complete the matrix, values were estimated to fill in the matrix. A sample matrix and the resulting output are contained in Appendix C. In the model data input file, a linear increase in the SDLM costs was used. Once accurate data is available for the deferral rate matrix, the costs FROM SCOST can be used.

6. $CO_{g,y}$: There are many components associated with the operation of fleet aircraft including personnel, consumables (POL, training/maintenance expendables),

depot level maintenance (includes SDLM costs and therefore needs to be adjusted), and sustaining investment (spare replenishment, training/support equipment maintenance, and software support). Depending on which of the above aspects are of interest, the CO value can be structured to include all or only some of them. To make the model as flexible as possible, the ability to designate costs for different blocks of aircraft cohort year groups was provided. This allows the differentiation between certain models and/or production techniques used when aircraft were produced. Since the operating and maintenance costs are presently maintained as an aggregate for the fleet and not broken down by model type or BUNO, a subjective decision must be made as to the magnitude of each of these costs for each group of aircraft cohorts.

In the model, the CO costs were adjusted for each cohort group so that no two groups would have exactly the same operating costs. This avoids having the solver attempt to differentiate between two "equal" alternatives and creating redundant solutions.

7. CR: The cost of retiring aircraft to the desert or selling them to foreign governments is nominal when compared to the other costs, but is included for completeness.

8. CT: As with the CR data, the cost of transferring an aircraft to the reserves is nominal when compared to the other costs, but is included for completeness.

9. BMAXP_t: The MPA budget is comprised of three primary components (APN1, APN5, O&S costs) and is assumed to have a minimum budget level of \$0. The BMAXP coefficients combine the APN1 and APN5 budgets since the avionics update line and the new airframe production lines are dependant on each other for production levels and other factors. This will allow for maximum flexibility and accommodate fluctuations in production/update levels to meet requirements. The maximum budget level is elasticized to allow for deviations above the BMAXP ceiling. The benefits for a deviation must overcome the penalty parameter **PABO_t**, which deters frivolous excursions above the maximum level, but allows deviations when the long term savings warrant them.

10. BMAXO_t: A separate budgetary limit on O&S funds is included, with its associated penalty variable **PABO_t**, to ensure the aircraft modernization program does

not just address the upgrade costs. The real costs of maintaining an aging fleet can be of additional concern. The model can easily accommodate adjustments in the coefficients based on specific cohort groups which have been diagnosed as having potential maintenance problems, as well as by model type when attempting to maintain equipment which is no longer in production.

B. PRODUCTION LINE DATA

The production line data is fairly self explanatory. The minimum and maximum levels of production are contractual limitations and correspond to those stated in the corporate proposals. These levels are presumed to represent maximum rate achievable with existing tooling and the minimum efficient rate. Levels outside this range, will incur penalties for opening a second line or keeping idle workforce on payroll. These penalties may include contractually negotiated payments in addition to other program incurred expenses. The minimum/maximum production level constraints have been elasticized by a variable with a penalty associated with it. The penalty can reflect the cost idle a workforce for a portion of the year or to open another production line. To account for the time it takes to ramp up to full production capability, the minimum/maximum levels are provided in terms of years from the initial procurement year (production campaign year y), versus calendar years t .

1. **PMIN_y**: Minimum new airframe production quantity level with associated elastic variable **BP_t** and penalty coefficient **PBP_t**. The PBP penalty coefficient can be used to impose a penalty for completely idling a plant for a year by producing 0 and incurring a penalty of PMIN times PBP. The penalties are associated with a given calendar year, not with the campaign year y .

2. **PMAX_y**: Maximum production level with associated elastic variable **AP_t** and penalty coefficient **PBP_t**.

3. **PMINREQ_y**: In the event contractual requirements are already in place, the minimum production levels required by year y of the contract would be inserted. The penalty variable **PB** and associated coefficient **PPB** are used to reflect the cost associated with breaking the contractually negotiated minimum production levels. This cost is the same whether the contract is broken once or ten times. If a what-if scenario

is desired, the **PPB** value can be set to zero and the model will ignore the cumulative minimum constraints.

4. **UMIN_{a,y}**: Minimum avionics update kit production quantity level with associated elastic variable **BU_t** and associated coefficient **PBU_t**.

5. **UMAX_{a,y}**: Maximum avionics update kit production quantity level with associated elastic variable **AU_t** and associated coefficient **PBU_t**.

6. **UMINREQ_{a,y}**: In the event contractual requirements are already in place, the minimum production levels required by year *y* of the contract would be inserted. **PU** and **PPU** are the associated elastic penalty variable and coefficient.

7. **LAG_t**: A lag time parameter is included since an aircraft is not rolled off the line in the year it is paid for. For the purposes of this model, the coefficient must be specified as an integer number of years and to allow flexibility, can be specified for each cohort year group.

C. INVENTORY RELATED DATA

All of the data is subjectively determined to satisfy the perceived needs of the future MPA fleet.

1. **IMIN_{t,r}**: Minimum inventory level required to meet operational needs. This figure does not include pipeline aircraft, which allow replacements to squadrons sending aircraft to SDLM and full aircraft allocation to the training squadrons. This required level is elasticized by the variable **BI_{t,r}** and associated penalty coefficient **PBI_{t,r}**.

2. **IMAX_{t,r}**: Represents the level which satisfies both the operational and pipeline requirements. This required level is elasticized by the variable **AI_{t,r}** and associated penalty coefficient **PAI_{t,r}**.

3. **H_{t,a}**: The definition of what is high technology is fairly arbitrary. The definition here implies the aircraft can handle all the threats posed in year *t*. As opposition technology improves, older systems become "obsolete" when considered against the newest threat, while still being able to accomplish their mission against older opposition variants. Since the model requires the parameter to be a binary digit, a gradual degradation of capability is not possible.

4. $HMIN_{t,r}$: Since the mission requirements and goals may be different in the USN and USNR, each service will set its own goal for the minimum percentage required to be "high tech". This value may change over time, and therefore HMIN is defined as the minimum percentage of fleet required to be "high tech" aircraft, in service type r , in year t . The HMIN constraint is elasticized by the variable $BH_{t,r}$ and associated penalty coefficient PBH_t . The PBH coefficients are set by year to allow different penalty weights for each year, allowing deviations for some years to be less severe than others. The HMIN values should take into account the expected future threat and the avionics presently installed and those envisioned for upgrades/new production aircraft. If the avionics can be easily upgraded by software modifications, consideration may be given to extending the period during which the avionics is considered "high tech".

5. $AMEAN_t$: To prevent the model from minimizing costs by simply not producing aircraft and letting the fleet reach a critical point after the model period ends, a maximum mean age limit is imposed. As with the HMIN data, AMEAN is set for each service type in a given year and is elasticized by the variable AB_t and associated penalty coefficient PAB_t . The AMEAN values should take into account the expected lifecycle for present inventory and new production aircraft.

6. $AMAX_t$: Each aircraft has a perceived useful life, based on stress, fatigue and corrosion. P-3s, produced since 1962, are just now approaching the end of their original service life, and therefore, detailed knowledge as to how long the aircraft can safely be flown is not available. An upper age limit of 40 years was chosen arbitrarily to limit the size of the problem. If certain groups of aircraft are known to possess structural deficiencies, this maximum age limit can be adjusted.

7. FTR_t : Each service type has a different operational tempo and therefore a different mean utilization flight time rate. This factor is used to project accumulated flight time for aircraft groups and aid in determining when mandatory SDLMs are to begin.

8. $FTMAX_t$: The maximum designed flight time for a cohort year group. If this limit is exceeded, the aircraft must undergo a SDLM corresponding to the one performed at 30 years of age. This SDLM must be repeated every 40 months until the

aircraft is retired. The aircraft would normally have to undergo these SDLMs after reaching the age of 30 years. The additional expense would therefore occur from the time the aircraft exceeds the maximum flight ceiling until it reaches 30 years of age. To account for this and still keep the problem a linear optimization model, an indicator variable $OF_{g,t}$ is used to trigger an additional expense of one third the mandatory SDLM cost (which corresponds to the SDLM conducted at the 30 year point) for every year after the aircraft exceeds this limit, until it reaches 30 years of age. This will overestimate the cost since the aircraft will already have an expected value for SDLMs in those years.

D. PENALTY VARIABLE COEFFICIENTS

The X-system elasticizes each constraint with a penalty variable. In each model there are constraints which are considered inviolable. These constraints are assigned a penalty coefficient which is so large, that violation of the constraint indicates some serious problems with either the model, or the input data set. The magnitude of the coefficient must however be kept as small as possible to avoid numerical difficulties within the solver. It should be set such that it is a magnitude larger than other penalty coefficients and the cumulative effects of other constraints do not make it advantageous to violate the "inviolable". In this model, the values for these inviolable penalty coefficients are set at 5900. In all model runs to date, this has been sufficient to avoid any constraint violations. The violatable constraints must have their penalty coefficients accordingly.

The modernization solution will probably not be possible without the violation of at least one of the High Tech, Mean Age, Budget, or Inventory constraints. The values assigned to each of the penalty variables determine the relative priority given to each of the constraints and therefore which are violated first.

Since the minimum inventory constraints dictate a level below which assigned missions may not be accomplished, this should be given the highest priority. This constraint represents one which is not inviolable in that it indicates infeasibility if violated, should nevertheless be violated only as a last resort. The PBI penalty coefficient is therefore set a magnitude lower than the "inviolables", but above other

penalty coefficients. Additionally, there are two different degrees of importance for the constraint, depending on whether the constraint deals with the USN or USNR. It could be argued the USN has higher priority due its peacetime operational commitments, and therefore should have a higher minimum inventory violation penalty. Values of 400 and 300 for $PPBI_{USN,t}$ and $PPBI_{USNR,t}$ respectively, were sufficient to avoid violations in model runs.

In order not to violate the minimum inventory constraints, the remaining penalty coefficients must be small enough such that their cumulative effects do not exceed the **PPBI** penalty. Concurrently, they must be large enough so the penalty is larger than the cost to satisfy the constraint (i.e., buy another aircraft, update kit).

A unit of violation for the budget constraint is self explanatory. A unit exceeding the limit is a million dollars over budget. The penalty has the affect of magnifying the amount over budget by a factor of **PABP/PABO**, depending on the type of budget constraint. A unit of violation for the high tech and mean age constraints is a little more ambiguous.

In the high tech constraint, a unit of violation requires the upgrade of one of the "non-high tech" aircraft. The worst case cost for this would be the purchase of one new production and the retirement of one non-high tech aircraft. This is essentially the value for **CP**. The value for **PBH** should therefore be close to the value of **CP**.

In the mean age constraint, a unit of violation corresponds to one aircraft being over **AMEAN** by one year. To correct this situation and still maintain the same inventory level, requires the purchase of $(1/AMEAN)$ new production and the retirement of the overaged aircraft. In other words, for every **AMEAN** units of violation, a new production aircraft must be purchased. The value for **PAA** should therefore be close to $(1/AMEAN)*CP$.

The priority decision is critical in determining the values for the **PABP**, **PABO**, **PBH**, **PAA** penalties. If the budget penalty is set too low, the high tech and mean age constraints will be satisfied by purchasing/upgrading more aircraft and exceeding the maximum budget limit. In the input data set used in this model run, a priority system of budget, high tech, then mean age was used. The budget and high tech penalties were valued to make them approximately equivalent, with the mean age being the

preferred constraint to violate. Values of 5, 50, 4 were used for the USN/USNR penalty coefficients for PABP, PBH, PAA respectively. The current model version does not set an explicit budgetary limit on O&M expenses and therefore PABO is set to zero (The mechanism is in place to set a O&M maximum budget, but was not used).

E. SAMPLE DATA SET

The data set below was used for the model runs discussed in this thesis. The "*" in the first character space are used to identify the line as a comment line in FORTRAN.

```
*              INPUT DATA SET
* The following data set is used as a front end loader for the
* EDITOR program, which will generate the matrices mentioned
* above. It is important that the format for this data set remain
* EXACTLY as presented, since the Editor program expects the
* entries to be in specific locations and in a particular format.
* Each space for data insertion is coded as to the type of value
* which can be used (I=integer, F=real number, X=character). The
* number of letters indicates the space allowed for the entry.
*
*
&TITLE
*
*              *** max of 10 72-character lines
*
*****
*              MPA PROCUREMENT MODEL
*              (USING PHOENIX BASE MODEL)
*
* INPUT DATA DATE:
* RUN DATE:
*
* SCENARIO: 20-YR PLAN (1991-2010)
*
*****
*
*
```

&TIME

*

* Defines the first and last planning year for this study (in the
* range 1991-2015), and maximum possible number of aircraft in each
* cohort year group (maximum number = 20).

*

PLANNING YEAR		MAX
FIRST	LAST	GRP NUM
IIII	IIII	II
1991	2000	04

*

*

&CURRENT INVENTORY CHARACTERISTICS

*

* Provides current configuration of present MPA fleet. Data is provided
* by cohort year group (first four numbers = digits of cohort year,
* last two numbers = group number), and number in each group,
* followed by model type, whether aircraft is in USN or USNR (service
* type), and current accumulated flight time.

* NOTE: The cohort groups MUST be inserted in numerical order with a
* maximum of 200 groups.

*

COHORT		MODEL	SERV	FLIGHT
YEAR GROUP	NUM	TYPE	TYPE	TIME
IIIIII	II	XXXX	XXXX	IIII
196601	04	BMOD	USNR	17000
196602	03	BMOD	USNR	16000
196603	04	BMOD	USNR	15000
196604	04	BMOD	USNR	14500
196605	04	BMOD	USNR	13500
196701	04	BMOD	USNR	16000
196702	04	BMOD	USNR	15750
196703	04	BMOD	USNR	15500
196704	04	BMOD	USNR	15250
196705	04	BMOD	USNR	15000
196706	04	BMOD	USNR	14750
196707	04	BMOD	USNR	14500
196708	04	BMOD	USNR	14250
196709	03	BMOD	USNR	14000
196710	03	BMOD	USNR	13750
196801	03	BMOD	USNR	14750
196901	04	CU3	USN	14500
196902	03	CU3	USN	14000
196903	03	CU3	USN	14000
196904	03	CU3	USN	13500
197001	04	CU3	USN	14500
197002	04	CU3	USN	14000
197003	04	CU3	USN	14000
197004	04	CU3	USN	13500
197005	04	CU3	USN	13000

197006	01	CU3	USN	14000
197101	04	CU3	USN	13500
197102	04	CU3	USN	13000
197103	04	CU3	USN	12500
197104	04	CU3	USN	12000
197105	03	CU3	USNR	12500
197106	03	CU3	USNR	12000
197201	04	CU3	USN	13000
197202	04	CU3	USN	12500
197203	04	CU3	USN	12000
197301	04	CU3	USN	13500
197302	04	CU3	USN	12500
197303	04	CU3	USN	11500
197304	04	CU3	USN	11000
197305	04	CU3	USN	10500
197306	03	CU3	USN	10000
197401	04	CU3	USN	12000
197402	04	CU3	USN	11000
197403	04	CU3	USN	10000
197501	04	CU1	USN	11500
197502	03	CU1	USN	11000
197503	03	CU1	USN	10500
197504	03	CU1	USN	09500
197601	03	CU1	USN	11000
197602	03	CU1	USN	10000
197603	04	CU1	USN	09500
197701	04	CU1	USN	11000
197702	04	CU1	USN	10500
197703	03	CU1	USN	10000
197801	03	CU2	USN	10000
197802	03	CU2	USN	09500
197803	04	CU2	USN	09000
197901	04	CU2	USN	09500
197902	04	CU2	USN	08500
197903	04	CU2	USN	07500
197904	04	CU2	USN	06750
198001	04	CU2	USN	09000
198002	04	CU2	USN	08000
198003	04	CU2	USN	07000
198101	04	CU2	USN	08500
198102	03	CU2	USN	07500
198103	03	CU2	USN	05500
198104	03	CU2	USN	04500
198201	04	CU2	USN	07000
198202	04	CU2	USN	06000
198203	04	CU2	USN	05000
198301	03	CU2	USN	06500
198302	03	CU2	USN	05500
198303	03	CU2	USN	04500
198401	04	CU3	USN	06000

198402	03	CU3	USN	05000
198403	01	CU3	USNR	04000
198501	02	CU3	USN	05000
198502	03	CU3	USN	04000
198601	04	CU3	USN	04500
198602	03	CU3	USN	03500
198603	01	CU3	USNR	03000
198701	03	CU3	USN	04500
198702	03	CU3	USN	03500
198703	03	CU3	USNR	03000
198801	04	CU3	USNR	03500
198802	03	CU3	USNR	03000
198901	04	CU3	USN	02500

*
*

INVENTORY REQUIREMENTS IN THE FUTURE

*

* Provides, by fiscal year, the required inventory (min
* and max levels;IMIN/IMAX), and penalties for deviating
* above max inventory level (PAI), and below minimum inventory
* (PBI) and the anticipated FMS sales.

*

	IMIN		IMAX		PAI		PBI		
* YEAR	USN	USNR	USN	USNR	USN	USNR	USN	USNR	FMS
* IIII	III	III	III	III	FFFF	FFFF	FFFFF	FFFFF	II
1991	232	078	274	096	01.00	01.00	400.00	300.00	00
1992	232	078	274	096	01.00	01.00	400.00	300.00	00
1993	232	078	274	096	01.00	01.00	400.00	300.00	00
1994	232	078	274	096	01.00	01.00	400.00	300.00	00
1995	232	078	274	096	01.00	01.00	400.00	300.00	00
1996	232	078	274	096	01.00	01.00	400.00	300.00	00
1997	232	078	274	096	01.00	01.00	400.00	300.00	00
1998	232	078	274	096	01.00	01.00	400.00	300.00	00
1999	232	078	274	096	01.00	01.00	400.00	300.00	00
2000	232	078	274	096	01.00	01.00	400.00	300.00	00
2001	232	078	274	096	01.00	01.00	400.00	300.00	00
2002	232	078	274	096	01.00	01.00	400.00	300.00	00
2003	232	078	274	096	01.00	01.00	400.00	300.00	00
2004	232	078	274	096	01.00	01.00	400.00	300.00	00
2005	232	078	274	096	01.00	01.00	400.00	300.00	00
2006	232	078	274	096	01.00	01.00	400.00	300.00	00
2007	232	078	274	096	01.00	01.00	400.00	300.00	00
2008	232	078	274	096	01.00	01.00	400.00	300.00	00
2009	232	078	274	096	01.00	01.00	400.00	300.00	00
2010	232	078	274	096	01.00	01.00	400.00	300.00	00

*
*

&HIGH TECH AND MAX MEAN AGE LIMITS

*
 * Provides, by fiscal year, the desired high tech min
 * (HMIN) and penalty for violating minimum (PBH), maximum
 * desired mean age (AMEAN) and associated penalty (PAA).
 *
 *

	HMIN		PBH		AMEAN		PAA	
* YEAR	USN	USNR	USN	USNR	USN	USNR	USN	USNR
* IIII	FFFF	FFFF	FFFFF	FFFFF	FFFF	FFFF	FFFFF	FFFFF
1991	0.50	0.30	50.00	40.00	16.0	18.0	04.00	03.00
1992	0.55	0.30	50.00	40.00	16.0	18.0	04.00	03.00
1993	0.60	0.35	50.00	40.00	16.0	18.0	04.00	03.00
1994	0.65	0.35	50.00	40.00	16.0	18.0	04.00	03.00
1995	0.68	0.37	50.00	40.00	16.0	18.0	04.00	03.00
1996	0.72	0.40	50.00	40.00	16.0	18.0	04.00	03.00
1997	0.75	0.42	50.00	40.00	16.0	18.0	04.00	03.00
1998	0.50	0.20	50.00	40.00	16.0	18.0	04.00	03.00
1999	0.55	0.23	50.00	40.00	16.0	18.0	04.00	03.00
2000	0.60	0.25	50.00	40.00	16.0	18.0	04.00	03.00
2001	0.65	0.28	50.00	40.00	16.0	18.0	04.00	03.00
2002	0.70	0.31	50.00	30.00	16.0	18.0	04.00	03.00
2003	0.75	0.34	50.00	40.00	16.0	18.0	04.00	03.00
2004	0.80	0.37	50.00	40.00	16.0	18.0	04.00	03.00
2005	0.85	0.41	50.00	40.00	16.0	18.0	04.00	03.00
2006	0.90	0.50	50.00	40.00	16.0	18.0	04.00	03.00
2007	0.95	0.58	50.00	40.00	16.0	18.0	04.00	03.00
2008	1.00	0.65	50.00	40.00	16.0	18.0	04.00	03.00
2009	1.00	0.72	50.00	40.00	16.0	18.0	04.00	03.00
2010	1.00	0.80	50.00	40.00	16.0	18.0	04.00	03.00

* * &BUDGETARY LEVELS ANTICIPATED *

* Provides the anticipated APN (BMAXP) and O&S (BMAXO)
 * budget levels and their associated penalty coefficients,
 * (PABP, PABO) for each planning year (Millions of dollars).
 * If the O&S Maximum budgetary constraint is not to be used, insert
 * zero's into the O&S BMAXO/PABO columns.
 *

* PLAN	APN		O&S	
* YEAR	BMAXP	PABP	BMAXO	PABO
* IIII	FFFFFF	FFFFF	FFFFFF	FFFFF
1991	0500.0	03.00	0000.0	00.00
1992	0500.0	03.00	0000.0	00.00
1993	0500.0	03.00	0000.0	00.00
1994	0500.0	03.00	0000.0	00.00
1995	0500.0	03.00	0000.0	00.00
1996	0500.0	03.00	0000.0	00.00
1997	0500.0	03.00	0000.0	00.00

1998	0500.0	03.00	0000.0	00.00
1999	0500.0	03.00	0000.0	00.00
2000	0500.0	03.00	0000.0	00.00
2001	0500.0	03.00	0000.0	00.00
2002	0500.0	03.00	0000.0	00.00
2003	0500.0	03.00	0000.0	00.00
2004	0500.0	03.00	0000.0	00.00
2005	0500.0	03.00	0000.0	00.00
2006	0500.0	03.00	0000.0	00.00
2007	0500.0	03.00	0000.0	00.00
2008	0500.0	03.00	0000.0	00.00
2009	0500.0	03.00	0000.0	00.00
2010	0500.0	03.00	0000.0	00.00

*

*

&LAST HIGH TECH YEAR

*

* Provides the possible MPA model types and the last year
* it is considered high tech.

*

* MODEL	LAST HIGH
* TYPE	TECH YR
* XXXX	IIII
BMOD	1990
CU1	1990
CU2	1990
CU3	1998
CU4	2010
P7	2010

*

*

&MISC PENALTIES AND COSTS

*

* Provides the remaining penalty weights for deviating from minimum
* cumulative new aircraft/update kit production quantity contractual
* obligations (PPB/PUB), retirement costs (CR), transfer (CT) costs
* and lag time for new production P-7 aircraft.

*

* PPB	PUB	CR	CT	LAG P-7
* FFFFF	FFFFF	FFFFF	FFFFF	II
200.0	150.0	0.110	0.110	02

*

*

&OPERATING AND MAINTENANCE COSTS (PART 1)

* Provides the costs to update a block of cohort year groups
 * to an U4 configuration (CU), maximum allowed flight time without
 * under going 30yr SDLM, lag time between procurement and delivery of
 * aircraft in group and the maximum allowable age for aircraft in
 * group. The block counter is used in the next section, to associate
 * appropriate operating and maintenance cost to each cohort group block.
 * Enter the beginning and ending cohort year group
 * numbers which represent contiguous blocks of common type aircraft.
 * IMPORTANT NOTE: Every cohort group designated above, MUST be covered
 * by one of the blocks, with the blocks being entered in numerical
 * sequence. Therefore, the first block should start with the earliest
 * cohort group number, and the last block should end with the last
 * cohort group number. A maximum of 7 different blocks may be entered
 * without having to modify the Editor Program.

BLK COUNTER	BEGIN GROUP NUMBER	ENDING GROUP NUMBER	FTMAX	LAG	AMAX
I	IIIIII	IIIIII	IIII	II	II
1	196601	196801	20000	01	40
2	196901	197403	20000	01	40
3	197501	198303	20000	02	40
4	198401	198901	20000	02	40

&USN MAINTENANCE AND OPERATING EXPENSES (PART 2)

* Provides the operating and maintenance expenses for each block by age
 * of aircraft. The block group numbers correspond to the blocks of cohort
 * groups designated in the previous section. Insert the values for P-7
 * operating expenses in P-7 column.
 * NOTE: If all block groups are not required, the remaining groups may be
 * disregarded, and if more groups are required, a program change will
 * be required.

AGE	P-7	BLOCK COUNTERS						
		BLK1	BLK2	BLK3	BLK4	BLK5	BLK6	BLK7
II	FFFFF	FFFFF	FFFFF	FFFFF	FFFFF	FFFFF	FFFFF	FFFFF
00	2.120	2.520	2.520	2.520	2.520			
01	2.150	2.550	2.550	2.550	2.550			
02	2.180	2.580	2.580	2.580	2.580			
03	2.210	2.610	2.610	2.610	2.610			
04	2.240	2.640	2.640	2.640	2.640			
05	2.270	2.670	2.670	2.670	2.670			
06	2.300	2.700	2.700	2.700	2.700			
07	2.330	2.730	2.730	2.730	2.730			
08	2.360	2.760	2.760	2.760	2.760			
09	2.390	2.790	2.790	2.790	2.790			
10	2.420	2.820	2.820	2.820	2.820			

11	2.450	2.850	2.850	2.850	2.850
12	2.480	2.880	2.880	2.880	2.880
13	2.510	2.910	2.910	2.910	2.910
14	2.540	2.940	2.940	2.940	2.940
15	2.570	2.970	2.970	2.970	2.970
16	2.600	3.000	3.000	3.000	3.000
17	2.630	3.030	3.030	3.030	3.030
18	2.660	3.060	3.060	3.060	3.060
19	2.690	3.090	3.090	3.090	3.090
20	2.720	3.120	3.120	3.120	3.120
21	2.750	3.150	3.150	3.150	3.150
22	2.780	3.180	3.180	3.180	3.180
23	2.810	3.210	3.210	3.210	3.210
24	2.840	3.240	3.240	3.240	3.240
25	2.870	3.270	3.270	3.270	3.270
26	2.900	3.300	3.300	3.300	3.300
27	2.930	3.330	3.330	3.330	3.330
28	2.960	3.360	3.360	3.360	3.360
29	2.990	3.390	3.390	3.390	3.390
30	3.020	3.420	3.420	3.420	3.420
31	3.050	3.450	3.450	3.450	3.450
32	3.080	3.480	3.480	3.480	3.480
33	3.110	3.510	3.510	3.510	3.510
34	3.140	3.540	3.540	3.540	3.540
35	3.170	3.570	3.570	3.570	3.570
36	3.200	3.600	3.600	3.600	3.600
37	3.230	3.630	3.630	3.630	3.630
38	3.260	3.660	3.660	3.660	3.660
39	3.290	3.690	3.690	3.690	3.690
40	3.320	3.720	3.720	3.720	3.720

*

*

&USNR MAINTENANCE AND OPERATING EXPENSES (PART 3)

*

* Provides the operating and maintenance expenses for each block by age of aircraft. The block group numbers correspond to the blocks of cohort groups designated in the previous section. Insert the values for P-7 operating expenses in P-7 column.

* NOTE: If all block groups are not required, the remaining groups may be disregarded, and if more groups are required, a program change will be required.

*

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BLOCK COUNTERS

AGE	P-7	BLK1	BLK2	BLK3	BLK4	BLK5	BLK6	BLK7
II	FFFFF	FFFFF	FFFFF	FFFFF	FFFFF	FFFFF	FFFFF	FFFFF
00	2.020	2.420	2.420	2.420	2.420			
01	2.050	2.450	2.450	2.450	2.450			
02	2.080	2.480	2.480	2.480	2.480			
03	2.110	2.510	2.510	2.510	2.510			
04	2.140	2.550	2.540	2.540	2.540			

05	2.170	2.570	2.570	2.570	2.570
06	2.200	2.600	2.600	2.600	2.600
07	2.230	2.630	2.630	2.630	2.630
08	2.260	2.660	2.660	2.660	2.660
09	2.290	2.690	2.690	2.690	2.690
10	2.320	2.720	2.720	2.720	2.720
11	2.350	2.750	2.750	2.750	2.770
12	2.380	2.780	2.780	2.780	2.780
13	2.410	2.810	2.810	2.810	2.810
14	2.440	2.840	2.840	2.840	2.840
15	2.470	2.870	2.870	2.870	2.870
16	2.500	2.900	2.900	2.900	2.900
17	2.530	2.930	2.930	2.930	2.930
18	2.560	2.960	2.960	2.960	2.960
19	2.590	2.990	2.990	2.990	2.990
20	2.620	3.020	3.020	3.020	3.020
21	2.650	3.050	3.050	3.050	3.050
22	2.680	3.080	3.080	3.080	3.080
23	2.710	3.110	3.110	3.110	3.110
24	2.740	3.140	3.140	3.140	3.140
25	2.770	3.170	3.170	3.170	3.170
26	2.800	3.200	3.200	3.200	3.200
27	2.830	3.230	3.230	3.230	3.230
28	2.860	3.260	3.260	3.260	3.260
29	2.890	3.290	3.290	3.290	3.290
30	2.920	3.320	3.320	3.320	3.320
31	2.950	3.350	3.350	3.350	3.350
32	2.980	3.380	3.380	3.380	3.380
33	3.010	3.410	3.410	3.410	3.410
34	3.040	3.440	3.440	3.440	3.440
35	3.070	3.470	3.470	3.470	3.470
36	3.100	3.500	3.500	3.500	3.500
37	3.130	3.530	3.530	3.530	3.530
38	3.160	3.560	3.560	3.560	3.560
39	3.190	3.590	3.590	3.590	3.590
40	3.220	3.620	3.620	3.620	3.620

*
*

&PRODUCTION AND UPDATE LINE CAPABILITIES

*
 * For each campaign year, provides the minimum and maximum new
 * aircraft/update kit production quantity levels (PMIN/PMAX; UMIN/UMAX)
 * and penalties associated with violating min levels (PBP; PBU)
 * (maximum level is assumed to be a hard requirement), and any minimum
 * cumulative production levels require to meet contractual obligations
 * (PMINREQ/UMINREQ). PBP and PUB are indexed by calendar year vice
 * campaign year, so the values entered below will correspond to the
 * "CAMP YEAR" column plus BYR.
 *

* CAMP	PMIN	PBP	PMAX	PMINREQ	UMIN	PBU	UMAX	UMINREQ
* YEAR								
* II	II	FFFFFF	II	III	II	FFFFFF	II	III
00	01	41.042	02	001	04	41.083	11	000
01	01	41.042	10	002	12	41.083	49	000
02	05	41.042	24	003	12	41.083	49	000
03	09	41.042	24	008	12	41.083	49	000
04	09	41.042	24	025	12	41.083	49	000
05	09	41.042	24	050	04	41.083	49	000
06	09	41.042	24	075	00	41.083	49	000
07	09	41.042	24	100	00	41.083	49	000
08	09	41.042	24	125	00	41.083	49	000
09	09	41.042	24	125	00	41.083	49	000
10	09	41.042	24	125	00	41.083	49	000
11	09	41.042	24	125	00	41.083	49	000
12	09	41.042	24	125	00	41.083	49	000
13	09	41.042	24	125	00	41.083	49	000
14	09	41.042	24	125	00	41.083	49	000
15	09	41.042	24	125	00	41.083	49	000
16	09	41.042	24	125	00	41.083	49	000
17	09	41.042	24	125	00	41.083	49	000
18	09	41.042	24	125	00	41.083	49	000
19	09	41.042	24	125	00	41.083	49	000

&LINE OPENING AND CLOSING POSSIBILITIES

*
 * Provides the earliest (EARL) and latest (LATE) possible
 * opening and closing years for the new aircraft and update
 * kit production lines and the minimum number of years a line must
 * be open before it can close.
 *

NEW AIRCRAFT					UPDATE KIT				
OPENING		CLOSING		MIN	OPENING		CLOSING		MIN
EARL	LATE	EARL	LATE	OPEN	EARL	LATE	EARL	LATE	OPEN
IIII	IIII	IIII	IIII	II	IIII	IIII	IIII	IIII	II
1992	1999	1998	2010	06	1991	1991	1998	2010	07

&FLIGHT TIME RATES

*
* Provides, by service type (USN/USNR), the flight time
* rate/year (FTR).
*

* SERVICE

* TYPE FTR
* XXXX IIII
* USN 0720
* USNR 0600
*
*

&SDLM COSTS

*
* Provides the SDLM costs depending on age of aircraft (include only
* those years where a SDLM is required). If some years have a zero cost
* they may be omitted.
*

* ACFT SDLM
* AGE COST
* II FFFFF
00 0.000
01 0.000
02 0.000
03 0.000
04 0.000
05 0.000
06 0.040
07 0.080
08 0.120
09 0.160
10 0.200
11 0.240
12 0.280
13 0.320
14 0.360
15 0.400
16 0.440
17 0.480
18 0.520
19 0.560
20 0.600
21 0.640
22 0.680
23 0.720
24 0.760
25 0.800
26 0.840
27 0.880
28 0.920

29	0.960
30	1.000
33	1.000
36	1.000

*
*

&UPDATE KIT UNIT PRODUCTION LINE COSTS

*

* Provides the per unit costs for the update kit (CU) production line.

*

*

*

CU
FFFFFF
10.000

*

*

&NEW AIRCRAFT UNIT PRODUCTION LINE COSTS

*

* Provides the per unit costs for the new aircraft (CP)
* production line. The CP value includes the price of the update kit.

*

*

*

NUMBER PURCHASED	CP
II	FFFFFF
01	45.000
02	44.700
03	44.400
04	44.100
05	43.800
06	43.500
07	43.200
08	42.900
09	42.600
10	42.300
11	42.000
12	41.700
13	41.400
14	41.100
15	40.800
16	40.500
17	40.200
18	39.900
19	39.600
20	39.300
21	39.000
22	38.700
23	38.400
24	38.100

*

*

&NONRECURRING COSTS

*

* Provides the non-recurring new aircraft and update kit production
* line costs for a campaign which begins in year 0. Pre-production
* costs should be preceded by a "-" sign (maximum of 6 pre-production
* years), followed by costs for years 0-5, and then place post-
* production costs in years 6-8 (maximum of 3 post-production years).

*

* CAMPAIGN	NEW AIRCRAFT	UPDATE KIT
* YEAR	COSTS	COSTS
* II	FFFFFFF	FFFFFFF
-6	000.000	000.000
-5	000.000	012.079
-4	000.000	045.830
-3	000.000	103.349
-2	063.452	099.908
-1	184.003	092.287
00	201.128	010.659
01	157.068	004.230
02	116.312	000.000
03	065.587	000.000
04	000.000	000.000
05	000.000	000.000
06	013.274	013.274
07	000.598	000.598
08	000.000	000.000

*

*

&TOOLING AMORTIZATION

*

* Provides for an avenue to amortize the new aircraft production
* tooling costs over life of campaign or 10 years, whichever is less.
* Amount entered will be distributed uniformly over this period.

*

* TOOLING COST
* FFFFFFFF
* 000.000

*

*

&END

APPENDIX C: SDLM EXPECTED COSTS

A. SDLM EXPECTED COST FORTRAN CODE

PROGRAM SCOST

- * This program computes the expected cost of a SDLM for an acft.
- * The values for a specific age are determined by calculating the probability an aircraft will require a SDLM at a given age.
- * This is determined by the deferral rates provided by the user.
- * Starting with the first ASPA, if an aircraft fails an ASPA, it must undergo a SDLM, and the Scost is determined as {P(FAIL that particular ASPA) * cost of that SDLM}.
- * The P(FAIL) values are determined by considering all the possible combinations of passing and failing previous ASPA's.
- *
- * The program allows for a user defined number of deferrals before the aircraft is required to undergo a SDLM (MXDEF).deferral rates for each of these must be specified for each SDLM cycle. The program allows for a total of 7 SDLM cycles and a maximum age of 30 years. The interval in years between each SDLM cycle and of a SDLM in a cycle must be specified in the declarations below.
- *

```
PARAMETER (MXDEF = 6, MXAGE = 30)
REAL COST(7)/.58,.62,.78,1.0,1.0,1.0,1.0/, SDLM(30),MULT,
+ DEF(0:5,7),P(0:6,7)
INTEGER A, S, D, DD,D2,D3,D4,D5,A1,A2,A3,A4,A5,
+ INTERV(7)/6,5,3,4,3,4,3/
```

```
DO 10 I=0,MXDEF-1
  READ (9,20) (DEF(I,J),J=1,7)
10 CONTINUE
20 FORMAT (7(F4.2,2X))
```

```
DO 30 A=1,MXAGE
  SDLM(A) = 0.0
30 CONTINUE
SDLM(30) = 1.0
```

```

DO 170 D=0,MXDEF
  S=1
  A1 = INTERV(S)+D
  IF (A1 .GE. MXAGE) GO TO 170
  P(D,S) = 1.0
  DO 40 DD=0,D-1
    P(D,S) = P(D,S) * DEF(DD,S)
40  CONTINUE
  IF (D.NE.6) P(D,S) = P(D,S) * (1.0-DEF(D,S))
  SDLM(A1) = SDLM(A1)+P(D,S) * COST(S)
  DO 160 D2=0,MXDEF
    S=2
    A2=A1+INTERV(S)+D2
    IF (A2 .GE. MXAGE) GO TO 160
    P(D2,S) = 1.0
    DO 50 DD=0,D2-1
      P(D2,S) = P(D2,S) * DEF(DD,S)
50  CONTINUE
    IF (D2.NE.6) P(D2,S) = P(D2,S) * (1.0-DEF(D2,S))
    SDLM(A2) = SDLM(A2)+P(D2,S) * COST(S) * P(D,S-1)
    DO 150 D3=0,MXDEF
      S=3
      A3=A2+INTERV(S)+D3
      IF (A3 .GE. MXAGE) GO TO 150
      P(D3,S) = 1.0
      DO 60 DD=0,D3-1
        P(D3,S) = P(D3,S) * DEF(DD,S)
60  CONTINUE
      IF (D3.NE.6) P(D3,S) = P(D3,S) * (1.0-DEF(D3,S))
      SDLM(A3) = SDLM(A4) +P(D3,S) * COST(S) * P(D2,S-1)
      DO 140 D4=0,MXDEF
        S=4
        A4=A3+INTERV(S)+D4
        IF (A4 .GE. MXAGE) GO TO 140
        P(D4,S) = 1.0
        DO 70 DD=0,D4-1
          P(D4,S) = P(D4,S) * DEF(DD,S)
70  CONTINUE
          IF (D4.NE.6) P(D4,S) = P(D4,S) * (1.0-DEF(D4,S))
          SDLM(A4) = SDLM(A4)+P(D4,S) * COST(S) * P(D3,S-1)
          DO 130 D5=0,6
            S=5
            A5=A4+INTERV(S)+D5
            IF (A5 .GE. MXAGE) GO TO 130
            P(D5,S) = 1.0
            DO 80 DD=0,D5-1
              P(D5,S) = P(D5,S) * DEF(DD,S)

```

```

80      CONTINUE
      IF (D5.NE.6) P(D5,S) = P(D5,S) * (1.0-DEF(D5,S))
      SDLM(A5) = SDLM(A5) + P(D5,S) * COST(S) * P(D4,S-1)
      DO 120 L6=0,MXDEF
        S=6
        A6=A5+INTERV(S)+D6
        IF (A6 .GE. MXAGE) GO TO 120
        P(D6,S) = 1.0
        DO 90 DD=0,D6-1
          P(D6,S) = P(D6,S) * DEF(DD,S)
90      CONTINUE
      IF (D6.NE.6) P(D6,S) = P(D6,S) * (1.0-DEF(D6,S))
      SDLM(A6) = SDLM(A6) + P(D6,S)*COST(S)*P(D5,S-1)
      DO 110 D7=0,6
        S=7
        A7=A6+INTERV(S)+D7
        IF (A7 .GE. MXAGE) GO TO 110
        P(D7,S) = 1.0
        DO 100 DD=0,D7-1
          P(D7,S) = P(D7,S) * DEF(DD,S)
100     CONTINUE
      IF (D7.NE.6) P(D7,S)=P(D7,S)*(1.0-DEF(D7,S))
      SDLM(A7)=SDLM(A7)+P(D7,S)*COST(S)*P(D6,S-1)
110     CONTINUE
120     CONTINUE
130     CONTINUE
140     CONTINUE
150     CONTINUE
160     CONTINUE
170 CONTINUE

      DO 180 A=1,MXAGE
        WRITE (10,*) A,SDLM(A)
180 CONTINUE

      STOP
      END

```


B. DEFERRAL MATRIX INPUT FOR PROGRAM

The deferral matrix provides the probability an aircraft will pass an ASPA on its j^{th} inspection in the i^{th} SDLM cycle (DEF_{ij}).

	j^{th} inspection						
i^{th}	0.92	0.77	0.64	0.5	0.5	0.5	0.5
SDLM	0.63	0.6	0.55	0.42	0.42	0.42	0.42
CYCLE	0.55	0.55	0.48	0.35	0.35	0.35	0.35
	0.45	0.4	0.38	0.3	0.3	0.3	0.3
	0.4	0.33	0.3	0.25	0.25	0.25	0.25
	0.3	0.25	0.2	0.15	0.15	0.15	0.15

C. FORTRAN PROGRAM OUTPUT

AGE EXPECTED COSTS

```
1 0.000000000E+00
2 0.000000000E+00
3 0.000000000E+00
4 0.000000000E+00
5 0.000000000E+00
6 0.463999882E-01
7 0.197431982
8 0.151275575
9 0.101690769
10 0.499209091E-01
11 0.347044170E-01
12 0.738019347E-01
13 0.112507403
14 0.740830779
15 0.476133764
16 0.904001117
17 0.702376008
18 0.631184280
19 0.620599508
20 0.616211116
21 0.387213111
22 0.828646719
23 0.642900229
24 0.569122076
25 0.557063758
26 0.552354574
27 0.301883638
28 0.771197140
29 0.600685120
30 1.00000000
```

APPENDIX D SAMPLE REPORT SUMMARY

The following is a report summary from a model run using a time horizon of five years and maximum group size of four.

YEAR	TOTAL INV	USN DESIRED		INVENTORY LEVELS					P7
		MIN	MAX	BMOD	AIRCRAFT CU1	MODEL CU2	TYPES CU3	CU4	
1991	210.0	212	274	0.0	30.0	62.0	108.0	10.0	0.0
1992	202.0	202	274	0.0	24.0	58.0	93.0	27.0	0.0
1993	202.0	202	274	0.0	13.0	48.0	77.0	64.0	0.0
1994	203.0	202	274	0.0	4.0	40.0	54.0	104.0	1.0
1995	202.0	202	274	0.0	0.0	36.0	29.0	130.0	7.0

YEAR	TOTAL INV	USNR INVENTORY LEVELS							
		DESIRED		BMOD	AIRCRAFT MODEL TYPES				P7
MIN	MAX	CU1	CU2		CU3	CU4			
1991	77.0	75	96	37.0	4.0	10.0	26.0	0.0	0.0
1992	73.0	70	96	25.0	4.0	13.0	30.0	1.0	0.0
1993	70.0	70	96	21.0	4.0	13.0	30.0	1.0	1.0
1994	70.0	70	96	21.0	0.0	13.0	26.0	9.0	1.0
1995	73.0	70	96	17.0	0.0	17.0	12.0	26.0	1.0

PRODUCTION LEVELS FOR NEW AIRCRAFT LINE STARTED IN 1993 AND STOPPED IN 1995

YEAR	LEVEL	LINE LIMITS		CUMULATIVE TOTAL	CONTRACT RQMT
		MIN	MAX		
1993	1.0	1	2	1.0	1
1994	1.0	1	10	2.0	2
1995	6.0	5	24	8.0	3

PRODUCTION LEVELS FOR UPDATE KIT LINE
STARTED IN 1991 AND STOPPED IN 1995

YEAR	----- KITS	LEVELS PROD	----- TOTAL	LINE MIN	LIMITS MAX	CUMM TOTAL	CONTRACT RQMT
1991	10.0	0.0	10.0	4	11	10.0	0
1992	18.0	0.0	18.0	12	49	28.0	0
1993	37.0	1.0	38.0	12	49	66.0	0
1994	48.0	1.0	49.0	12	49	115.0	0
1995	43.0	6.0	49.0	12	49	164.0	0

YEARLY SUMMARY OF INVENTORY ACTIVITY

YEAR	NEW ACFT PRODUCTION	ORIGINAL INVENTORY UPDATES	ACFT TRANSFERS	RETIREMENTS
1991	0.0	10.0	22.0	23.0
1992	0.0	18.0	8.0	12.0
1993	1.0	37.0	1.0	4.0
1994	1.0	48.0	0.0	0.0
1995	6.0	43.0	7.0	4.0

SUMMARY OF COHORT GROUP ACTIVITY

GRP#	GROUP	UPDATE	TRANSFER	RETIREMENT
1	196601	0	0	1991
2	196602	0	0	1991
3	196603	0	0	1991
4	196604	0	0	1991
5	196605	0	0	1991
6	196701	0	0	1991
7	196702	0	0	1992
8	196703	0	0	1992
9	196704	0	0	1992
10	196705	0	0	1993
11	196706	0	0	1995
12	196707	0	0	0
13	196708	0	0	0
14	196709	0	0	0
15	196710	0	0	0
16	196801	0	0	0
	...ONLY 1.000 OF GROUP 196901 UPDATED IN 1992			
	...ONLY 3.000 OF GROUP 196901 UPDATED IN 1993			
17	196901	1993	0	0
18	196902	1991	0	0

19	196903	1992	1995	0
20	196904	1991	0	0
	...ONLY 3.000 OF GROUP 197001 UPDATED IN 1992			
	...ONLY 1.000 OF GROUP 197001 UPDATED IN 1993			
21	197001	1993	0	0
22	197002	1994	1991	0
23	197003	1994	0	0
24	197004	1995	0	0
25	197005	1994	0	0
26	197006	1995	0	0
27	197101	1995	0	0
28	197102	1995	0	0
29	197103	1995	1991	0
30	197104	1995	1992	0
31	197105	1995	0	0
32	197106	1995	0	0
33	197201	1995	0	0
34	197202	1995	0	0
35	197203	1995	0	0
36	197301	1994	0	0
37	197302	1993	0	0
38	197303	1992	0	0
39	197304	1991	0	0
40	197305	1993	0	0
41	197306	1994	0	0
42	197401	1994	0	0
43	197402	1994	0	0
44	197403	1993	0	0
45	197501	1994	1991	0
46	197502	1992	0	0
47	197503	1993	0	0
48	197504	1994	0	0
49	197601	1994	0	0
50	197602	1992	0	0
51	197603	1993	0	0
52	197701	1995	0	0
53	197702	1993	0	0
54	197703	1994	0	0
55	197801	1993	0	0
56	197802	1993	0	0
57	197803	1993	0	0
58	197901	1994	0	0
59	197902	1994	0	0
	...ONLY 1.000 OF GROUP 197903 UPDATED IN 1992			
60	197903	1992	1991	0
61	197904	0	1995	0
62	198001	0	1992	0
63	198002	0	0	0
64	198003	0	0	0
65	198101	0	0	0

66	198102	0	0	0
67	198103	0	0	0
68	198104	0	1991	0
69	198201	0	0	0
70	198202	0	0	0
71	198203	0	0	0
72	198301	0	0	0
73	198302	0	0	0
74	198303	0	1991	0
75	198401	0	0	0
76	198402	0	0	0
77	198403	0	0	0
78	198501	0	0	0
79	198502	0	0	0
80	198601	0	0	0
81	198602	0	0	0
82	198603	0	0	0
83	198701	0	0	0
84	198702	0	0	0
85	198703	0	0	0
86	198801	0	0	0
87	198802	0	0	0
88	198901	0	0	0

APN1/5 BUDGETARY SUMMARY

YEAR	BUDGET		----- COMPONENT COSTS -----			
	USED	AUTH	PRODUCTION COSTS		FIXED LINE COSTS	
			NEW ACFT -	UPD KITS	NEW ACFT -	UPD KIT
1991	176.8	500.0	0.0	102.7	63.5	10.7
1992	374.6	500.0	0.0	186.4	184.0	4.2
1993	632.9	500.0	45.0	386.7	201.1	0.0
1994	701.9	500.0	45.0	499.9	157.1	0.0
1995	821.1	500.0	261.0	443.8	116.3	0.0

O&M BUDGETARY SUMMARY

YEAR	BUDGET		----- COMPONENT COSTS -----				
	USED	AUTH	INVENTORY		RETIRE	TRANS	FLT TIME
			OPERATING -	SDLM	-MENT	-FER	VIOLATION
1991	939.5	0.0	831.27	103.68	2.300	2.200	0.000
1992	908.2	0.0	801.52	104.72	1.200	0.800	0.000
1993	911.1	0.0	799.23	111.40	0.400	0.100	0.000
1994	930.9	0.0	809.48	121.44	0.000	0.000	0.000
1995	946.3	0.0	816.90	128.32	0.400	0.700	0.000

HIGH TECHNOLOGY SUMMARY

YEAR	USN		USNR	
	ACTUAL	REQUIRED	ACTUAL	REQUIRED
1991	0.56	0.50	0.34	0.30
1992	0.59	0.55	0.42	0.30
1993	0.70	0.60	0.46	0.35
1994	0.78	0.65	0.51	0.35
1995	0.68	0.68	0.37	0.37

MEAN AIRCRAFT AGE SUMMARY

YEAR	USN		USNR	
	ACTUAL	REQUIRED	ACTUAL	REQUIRED
1991	12.6	16.0	16.6	18.0
1992	13.6	16.0	16.2	18.0
1993	14.6	16.0	16.6	18.0
1994	15.5	16.0	17.5	18.0
1995	15.9	16.0	18.1	18.0

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